Új Nemzeti
Kiválóság Program

III. Nuclear models

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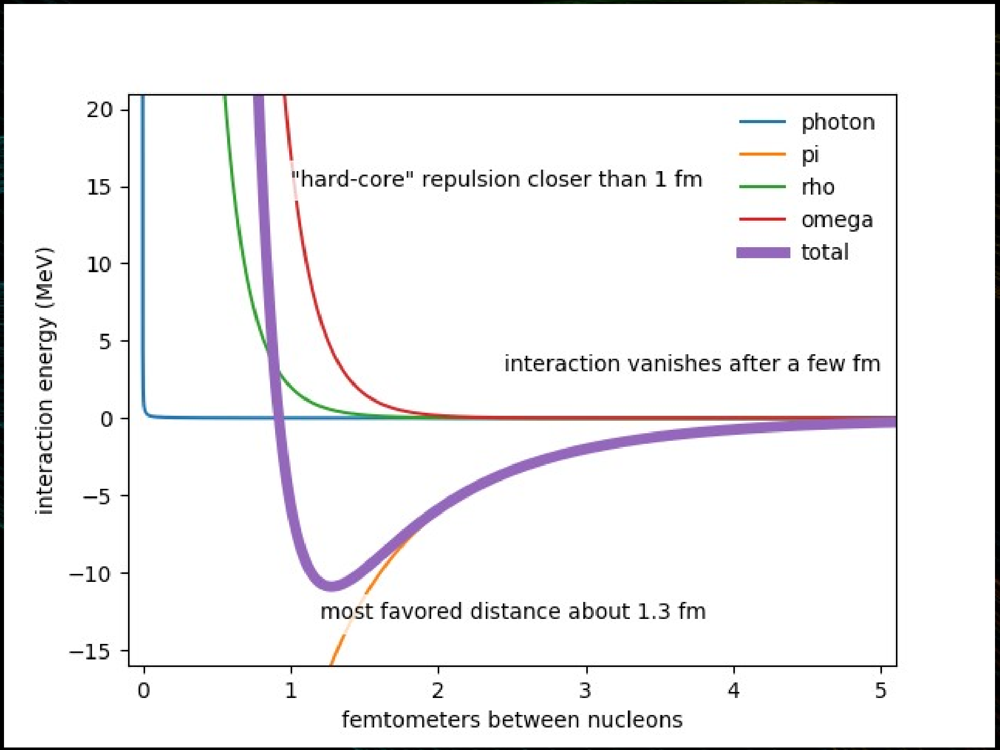
The nuclear „strong” interaction

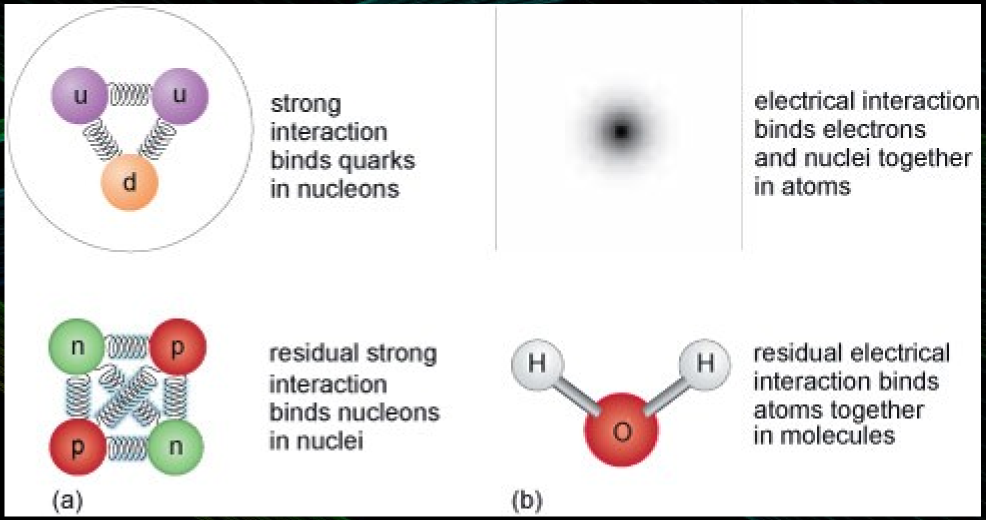
- Qualitative properties of nuclear interaction based on observations:
  - Short range strong attraction:
    - stable nuclei exist
    - the Rutherford scattering can be explained by the Coulomb-force
    - n-p scattering
  - Repulsive core:
    - saturation effect: 8 MeV/nucleon
  - Spin dependent:
    - parallel spins in deuteron
  - Charge independent:
    - level scheme of mirror nuclei
  - Non-central, tensor forces:
    - quadrupole moment of deuteron
  - Spin-orbit coupling:
    - splitting of energy levels

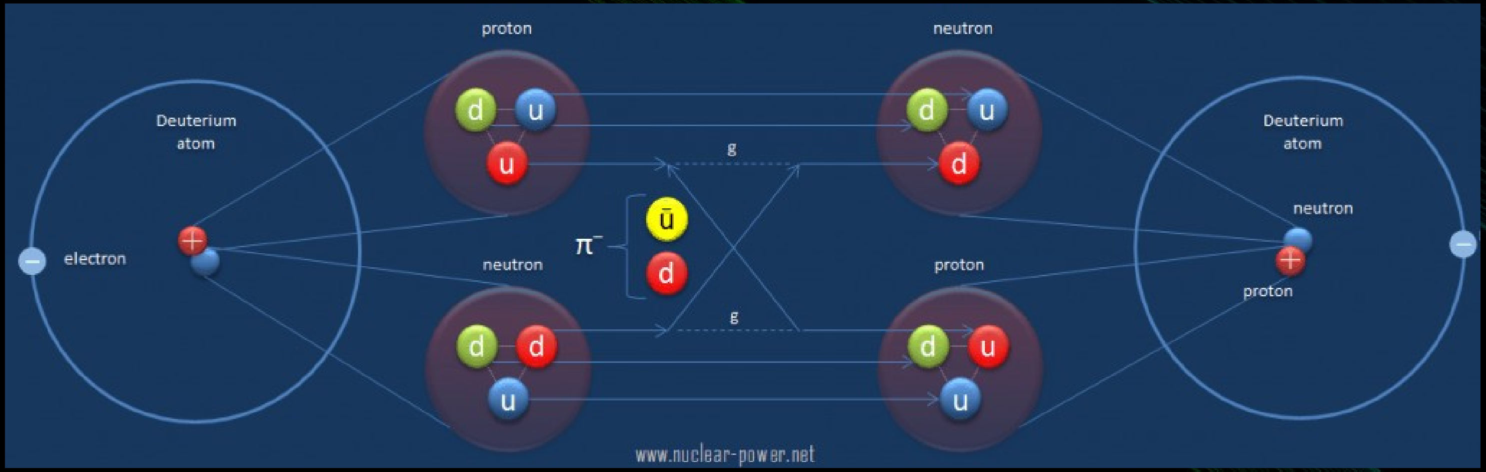


In contrast to the Coulomb interaction, the nuclear interaction has a very complicated form!!

The origin of the nuclear interaction

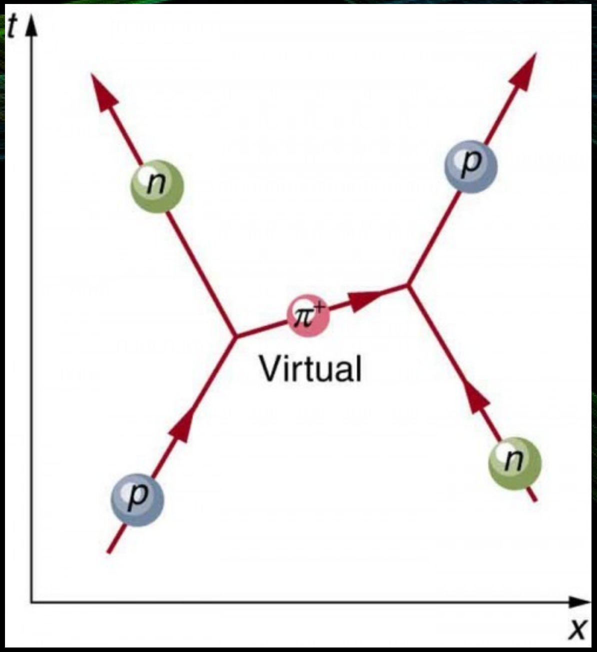
- The nuclear interaction is a Van der Waals type, effective interaction, a residual of the fundamental strong interaction between quarks!

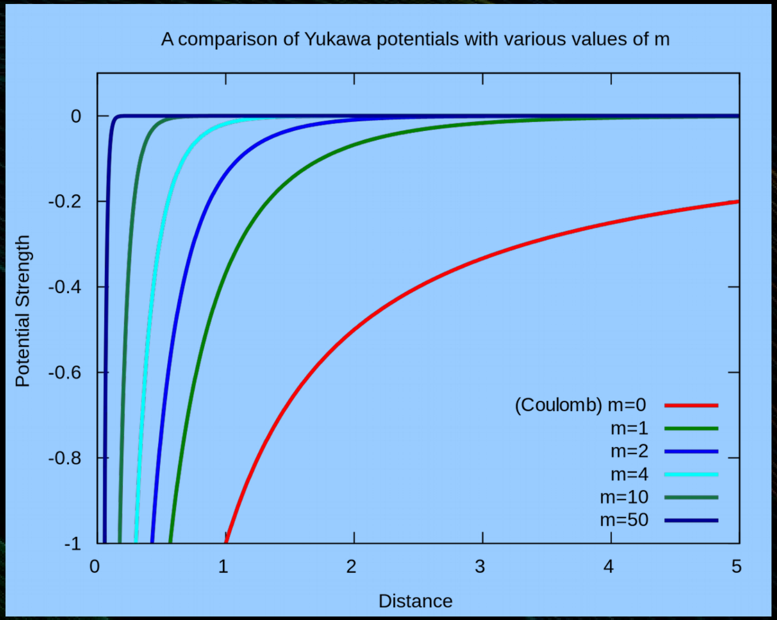




The origin of the nuclear interaction

- It can be approximated as a meson-exchange between nucleons
  - as electromagnetic interaction in QED: photon exchange between electric charges
  - but $m_{meson} > 0$ (not like m_{photon}) so the range of the interaction is finate!
  - Yukawa predicted π-mesons with $m_\pi=279m_e$





One-pion-exchange-potential (OPEP)

$$V_{OPEP} \sim g_{pi}^2 \left(\frac{m_\pi}{m_p}\right)^2 m_\pi c^2 \vec{\tau}_1 \cdot \vec{\tau}_2 [\vec{\sigma}_1 \cdot \vec{\sigma}_2 + S_{12} V_T] \frac{e^{-r/R}}{r/R}$$

$$S_{12} \equiv \frac{3}{r^2} (\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$V_T \equiv 1 + 3\frac{R}{r} + 3\frac{R^2}{r^2}$$

Yukawa potential

$$\phi = -g_N \frac{e^{-r/\lambda\!\!\!^{-}}}{r}, where\ \lambda\!\!\!^{-} = \hbar / mc$$

Why nuclear models?

- A bounded system is described by the wavefunction and the E energy which satisfies the time-independent Schrödinger equation:

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(\boldsymbol{x})\psi = E\psi$$

- Possible stationary states are discrete states having well-defined energy, spin and parity.
- Problem: the Schrödinger equation of a realistic nuclei cannot be solved, because
  - there are too many nucleons in a typical nucleus
  - the interaction is not precisely known, and very difficult
- Solution: using different models with different scope

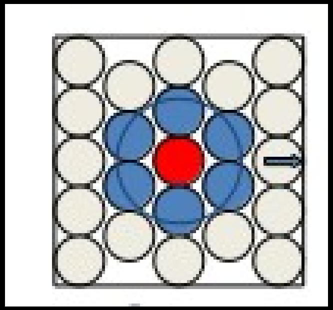
Liquid-drop model

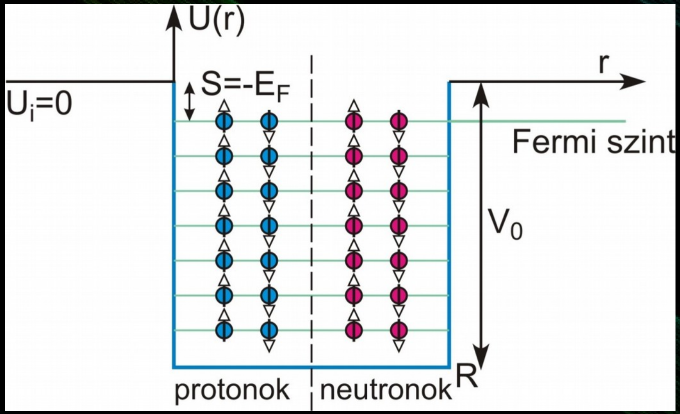
- Considering the results of charge distribution measurements, $r=r_0A^{1/3}$, which means nucleus is incompressible like a liquid! But also charged..
- $\rho = 10^{14}$ g/cm^3
- So the binding energy is built up from the following terms:
  - Volume term
  - Surface term
  - Coulomb term
  - Symmetry term
  - Pairing term

Weizsaecker emprical formula:

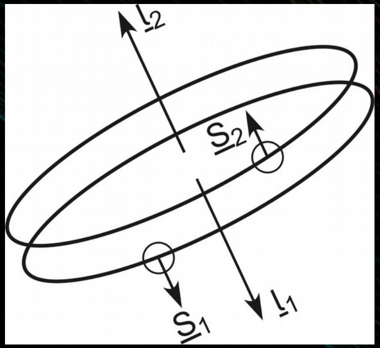
charged liquid drop | QM origin

$$\Delta W = \alpha A - \beta A^{2/3} - \gamma \frac{Z^2}{A^{1/3}} - \phi \frac{(A/2-Z)^2}{A} + \delta A^{3/4}$$





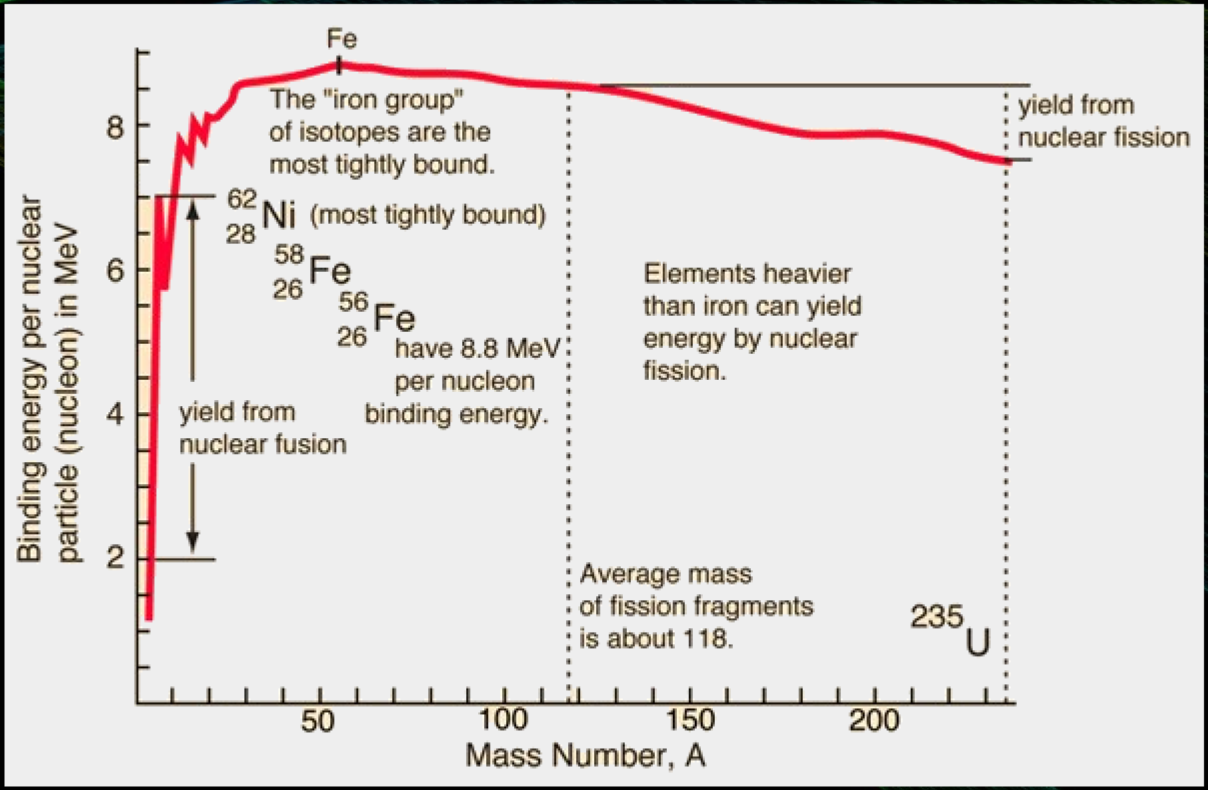
Pauli principle



Spin-dependency

LDM predictions

- Binding energies are quite OK → Masses, separation energies as well
- The nuclear fission process can be described
- Some properties of some specific excitations can be understood as vibrations of the nuclear surface: see next...



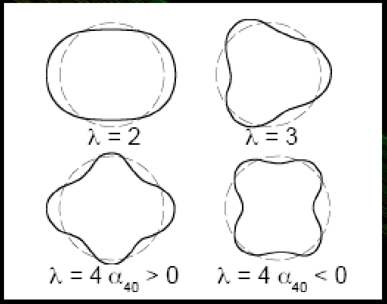
Vibrations in the LDM

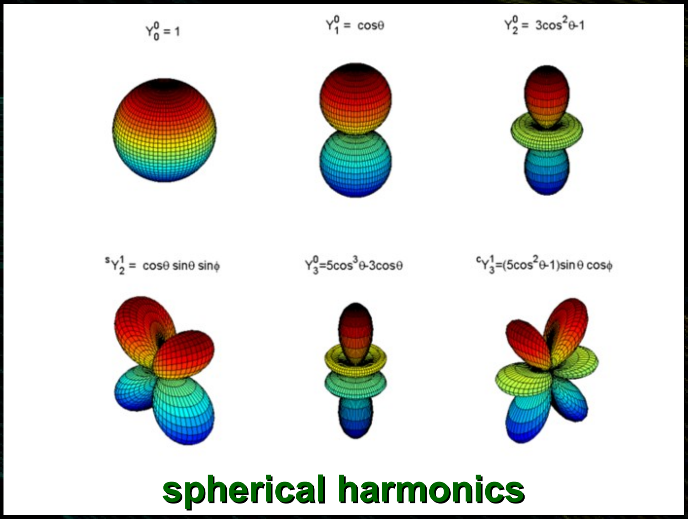
- Properties of vibrational states can be understood by LDM (level spacings, spin and parity): only even-even nuclei, vibrations around spherical shape:

$$R(t)=R_0\left[1+\sum_{\lambda=0}^{\infty}\sum_{\mu=-\lambda}^{\lambda}\alpha_{\lambda\mu}(t)Y_{\lambda\mu}(\theta,\phi)\right]$$

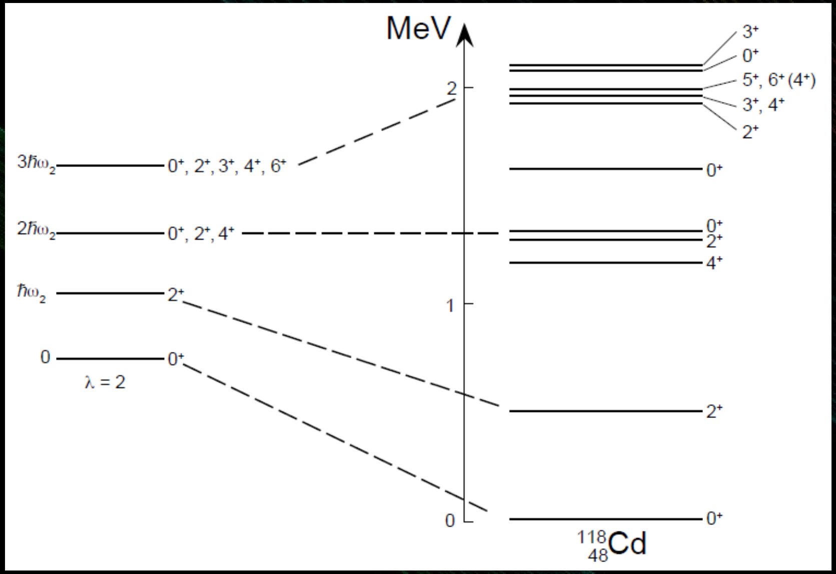
amplitude

spherical harmonics



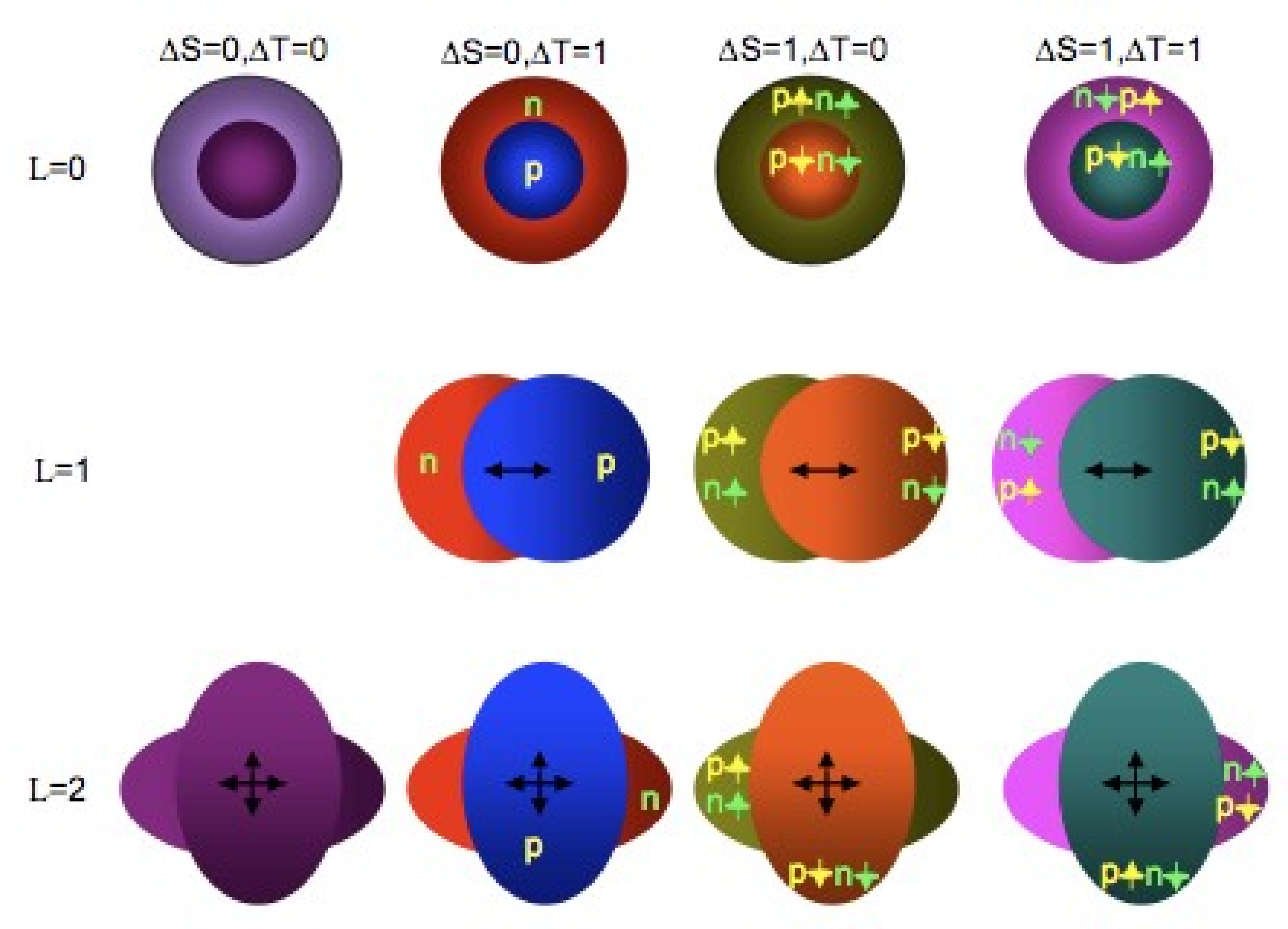


- Typical vibrational spectrum with equal energy spacings:
  - E=nhw   n=1,2,3...
- The frequency however, cannot be matched to the energy of the level!



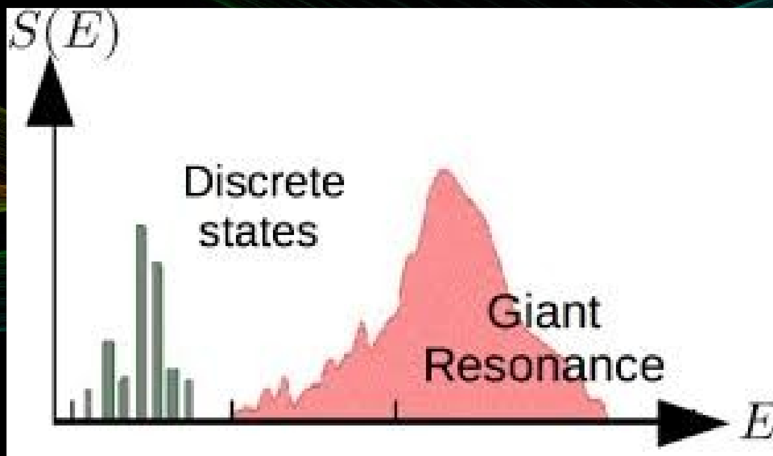
Giant resonances

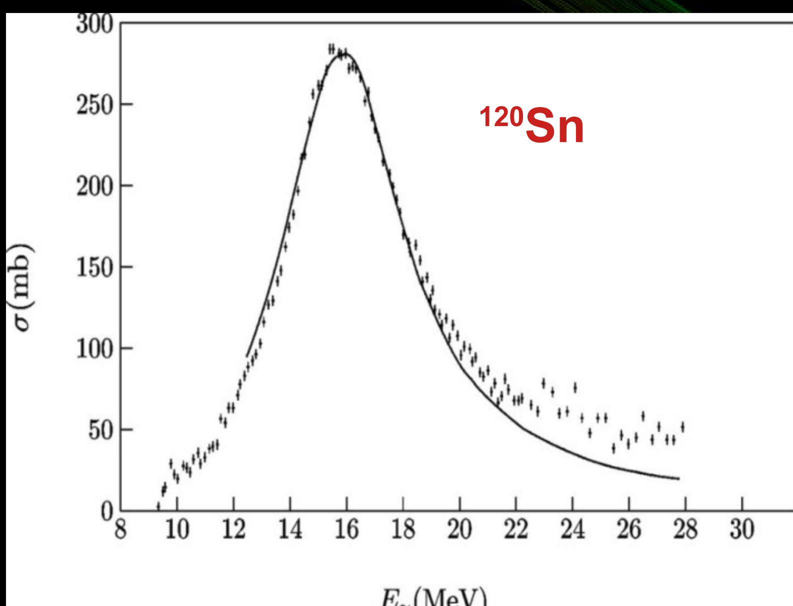
- Collective motions, small amplitude oscillations around the equilibrium shape and density (in LDM), where >50% of the nucleons participate
- Described by quantum numbers: spin (electric or magnetic), isospin (isoscalar or isovector), angular momentum (multipolarity)



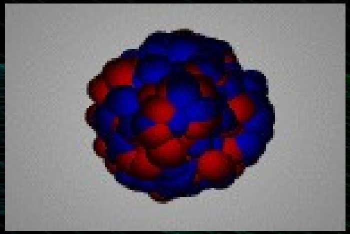
Giant resonances

- Historically, first observed in photoabsorption cross sections in ^{63}Cu

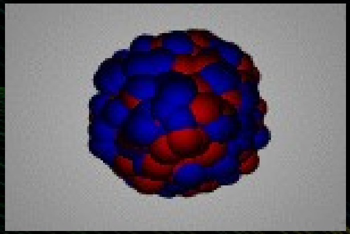




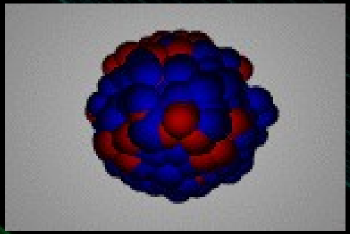
IVGMR



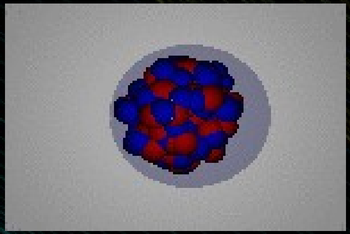
ISGMR



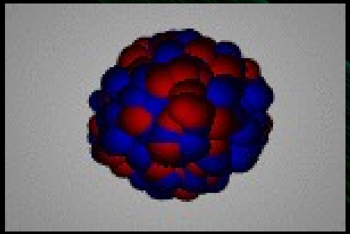
ISGQR



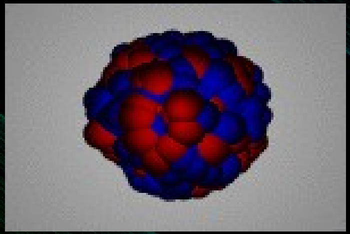
PDR



IVGQR

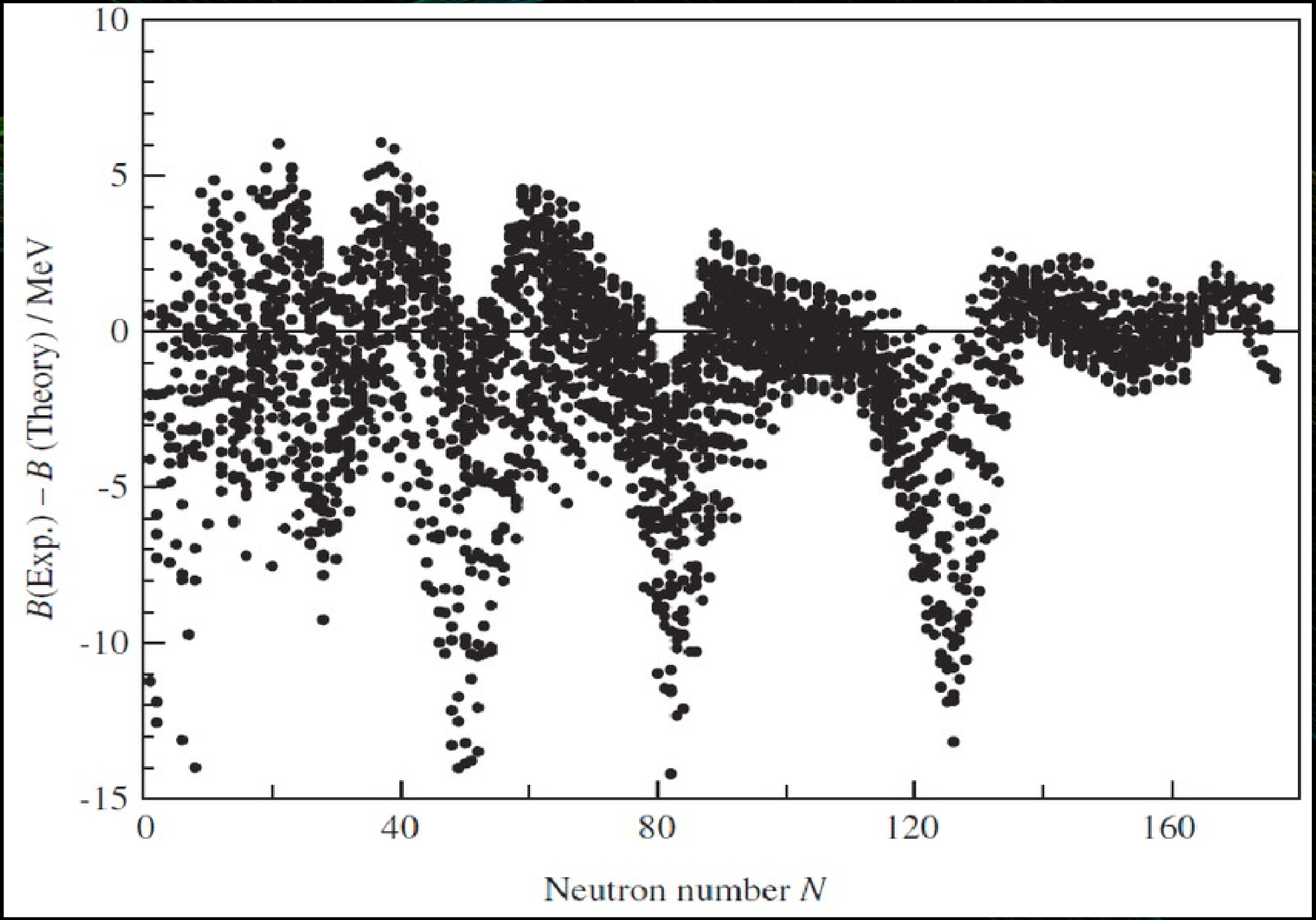


IVGDR



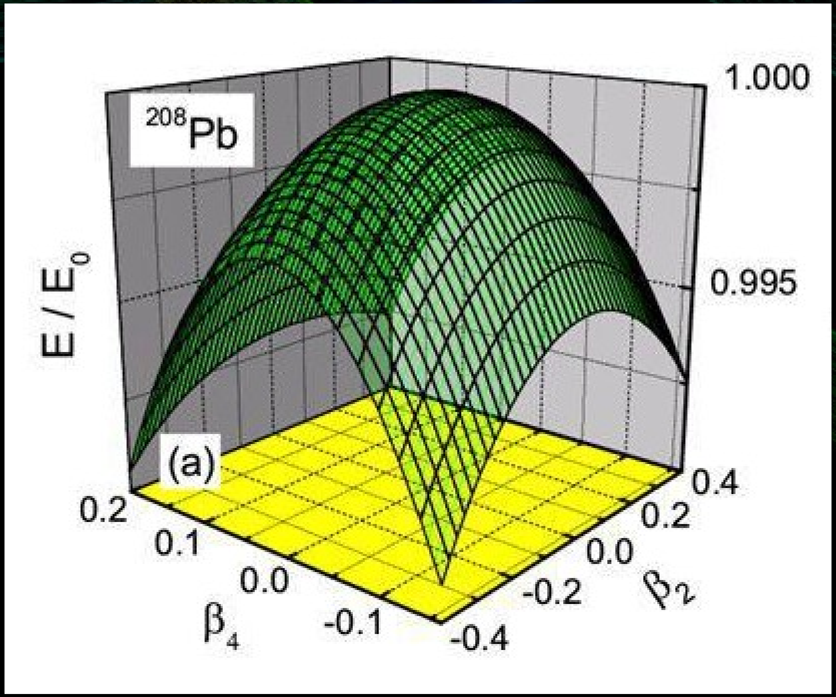
Problems with LDM

- **Magic numbers**: fine structure of the binding energies shows significant echancement of binding energy at Z or N=2,8,20,28,50,82,126 → indicating shell structure like in atoms

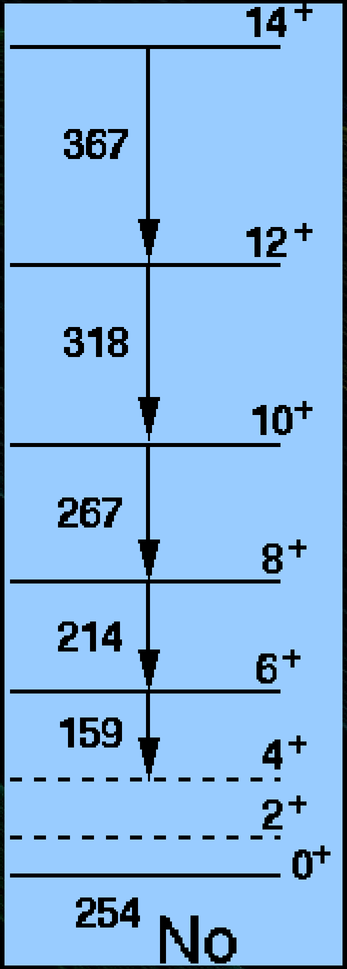


Problems with LDM

- Deformed nuclei exist (in ground state)! The energy minimum of the LDM is at zero deformation: spherical shape is favorable.



- Energy spectrum of many nuclei differs from vibrational



The Fermi-gas model

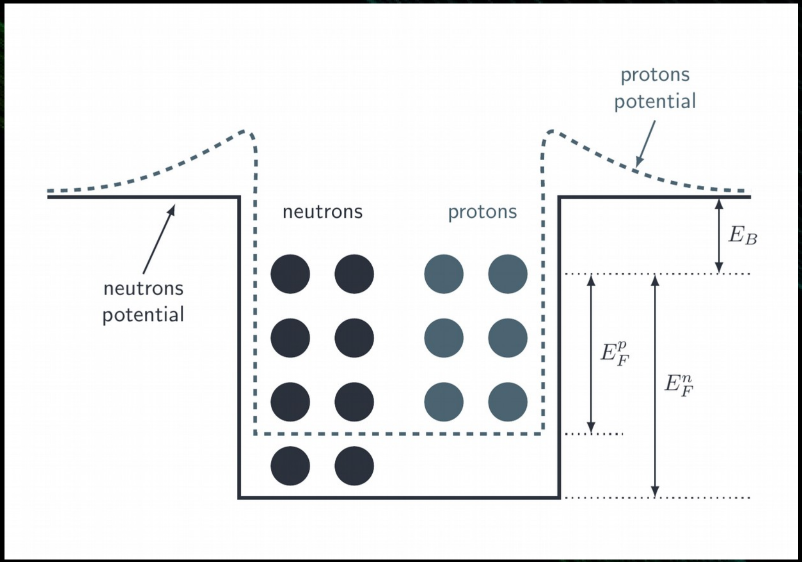
- LDM calculates only with potential(-like) energies depending on *R*
- But a particle in a confined space → kinetic energy due to the Heisenberg principle
- Independent particle model: not interacting praticles (fermions with s=1/2) in a spherical potential with radius $R=r_0 A^{1/3}$ and depth U_0 → Fermi – gas
- The only effect of other nucleons are the confine the nucleons in V

State density in Fermi statistics: (Pauli principle)

$$\frac{dN}{dp} = 2\frac{4\pi p^2 V}{(2\pi\hbar)^3}$$

p: momentum of nucleon
V: volume of nucleus

- Ground state of the nuclues → Fermi-gas at T=0, nucleons are at the deepest one-particle states
- $U_0=U_{kin}+U_{separation}$=32 MeV+8 MeV (if N>Z $U_{0(n)}>U_{0(p)}$ due to Coulomb potential)



The Fermi-gas model

- From calculations, the kinetic energy of the nucleus:
  - has volume, surface and symmetry terms
  - means that kinetic energy contributes to the liquid drop potential energies (α, β, φ)

$$E_A = c_v A + c_F A^{2/3} + c_s \frac{(N-Z)^2}{A}$$

- For excitation energies, where kinetic energy of nucleons expected to be larger, Fermi gas model is even more important!
- **Qualitative explanation of symmetry energy and saturation**

The nuclear shell model

- **Magic numbers**: fine structure of the binding energies shows significant echancement of binding energy and zero quadrupole moments (thus spherical shape) at Z or N=2,8,20,28,50,82,126→ indicating shell structure like in atoms
- Isotope abundance in nature and systematics of alpha and beta decay also points to shell structure in nuclei

- Shell model of atoms
  - central Coulomb potential
  - weakly interacting electrons

- Shell model of nuclei
  - non central potential
  - nucleons are strongly interacting

?? With these conditions, can we apply shell model at all ??

Approximations:

- But! Pauli principle → in the ground state, nucleons occupy the lowest single particle states → Nucleon nucleon scattering cannot really occur since energy exchange cannot happen due to the occupied states → the mean free path of nucleus is getting large → quasi independent nucleons!
- The effect of the nucleons (on a specific nucleon) by the very short-range interaction can be approximated by a central potential with spehrical symmetry

The nuclear shell model

- Nuclear potential

$$\rho_F(r)=\frac{\rho_0}{1+e^{\frac{r-c}{z}}}$$

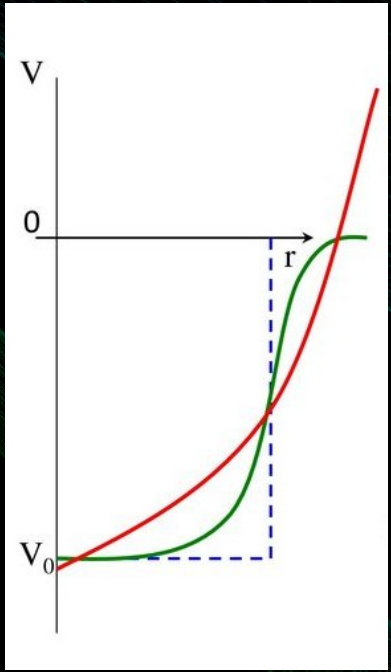
Experimental density of nuclear material
Fermi function

$$V(r)=-\frac{V_0}{1+e^{\frac{r-c}{z}}}$$

Realistic potential
Wood-Saxon potential

No analytical solution of Schrödinger equation with Wood-Saxon type

- Harmonic oscillator (for light nuclei) and square-well potential (for heavier nuclei) are good approximations

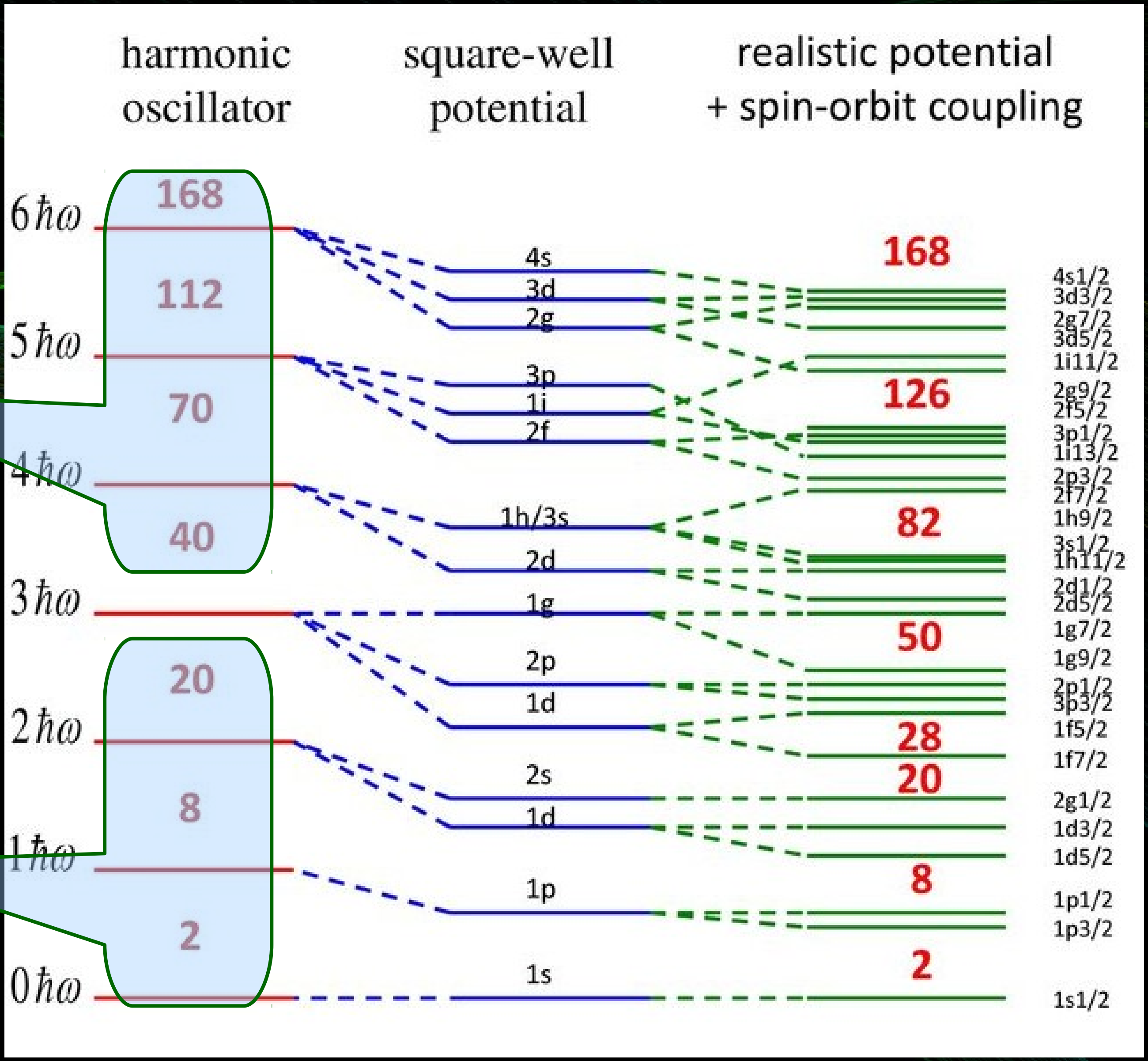


The nuclear shell model

Wood-Saxon gives the same as HO!!

These shell closures are **not** OK!!

These shell closures are OK!!



Nuclear shell model: spin-orbit interaction

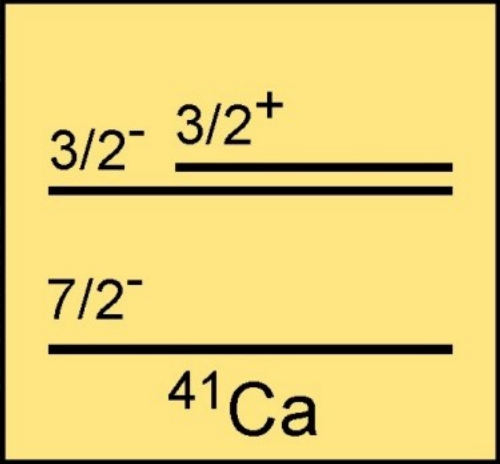
- For electrons, this interaction stem from the Dirac equation: magnetic moments of the spin motion and the orbital motion interacts, „couples”
- *Goepert-Mayer*: For nuclei spin-orbit coupling is not deduced from theory, the strength fitted to experiments
- For given angular momentum ***l*** two values depending on the relative direction of ***s*** and ***l*** :

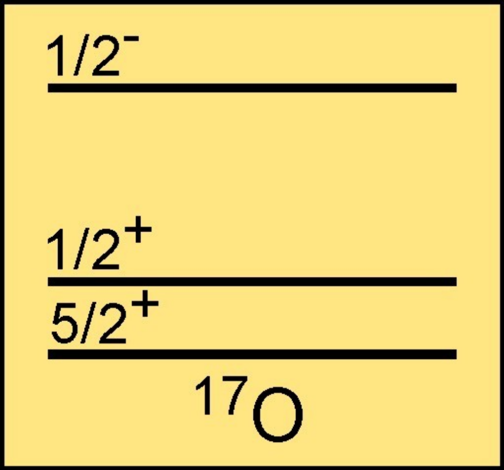
$$V = V(r) + U(r)(\vec{s}\,\vec{l})$$

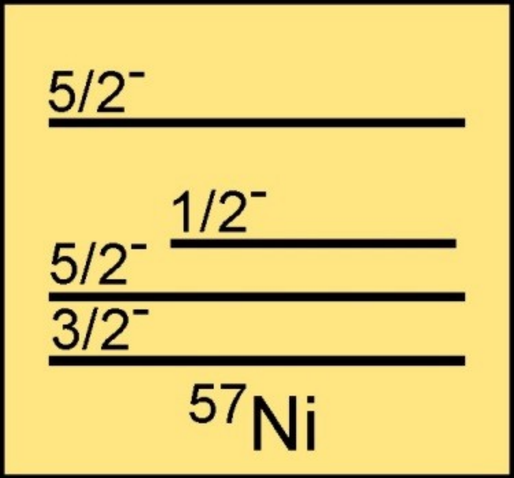
- So a level with given *l* splits to two levels with *i=l* ± ½
- Parallel spin and angular momentum → lower energy (higher interaction energy)
- Splitting is large for large *l* → for *l>4* the sub-levels are in different shells!
- Giving good magic numbers! (see previous slide)
- Far from stability magic number are different (8→6 and 20→16)

One-particle shell model

- A closed shell + one valence nucleon
- Gives (in most cases) right ground state spin and parity of spherical odd nuclei
  - total angular momentum is determined fully by the valence nucleon
- Gives right spins and parities of low excitated states of odd nuclei with spherical symmetry
  - hole excitations beside particle excitations
- But what happens if deformation is present?
  - Many valence nucleons can deform the mean field potential

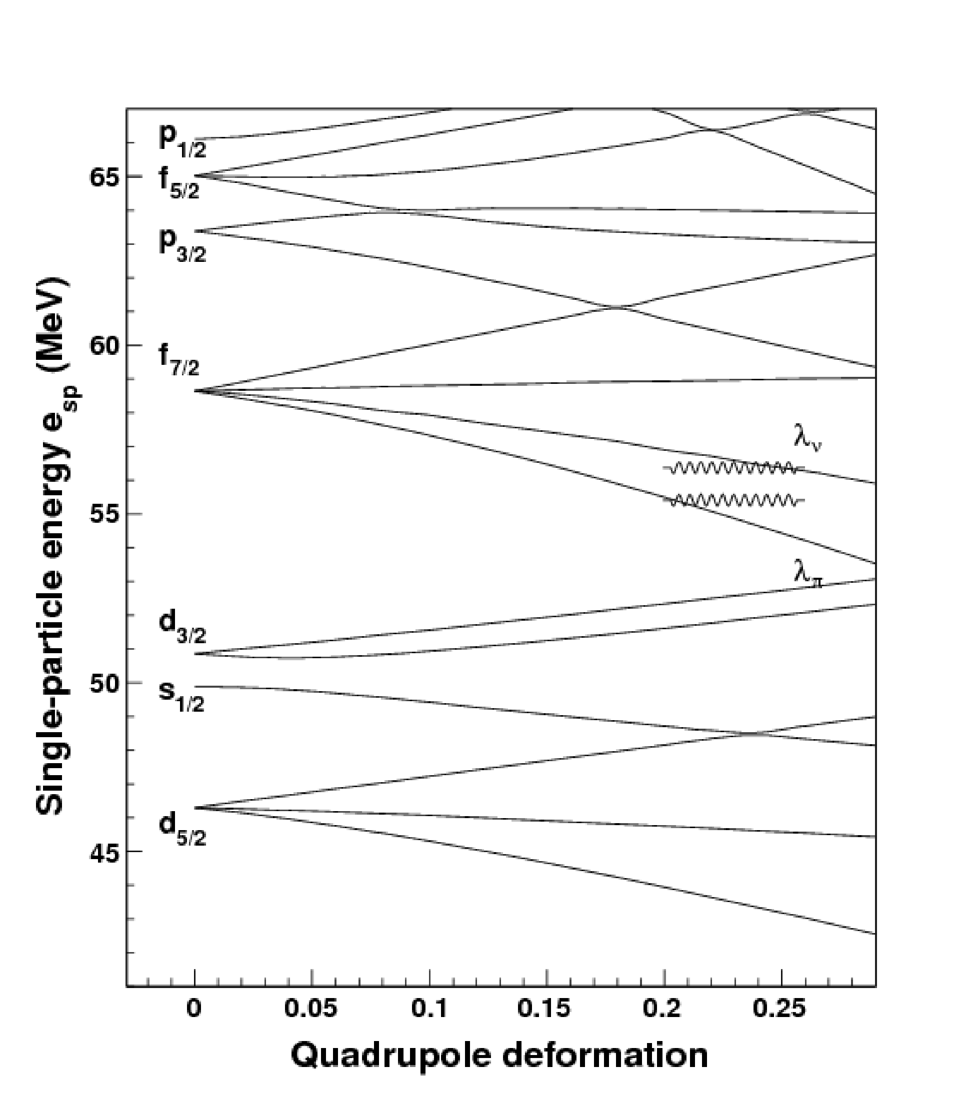




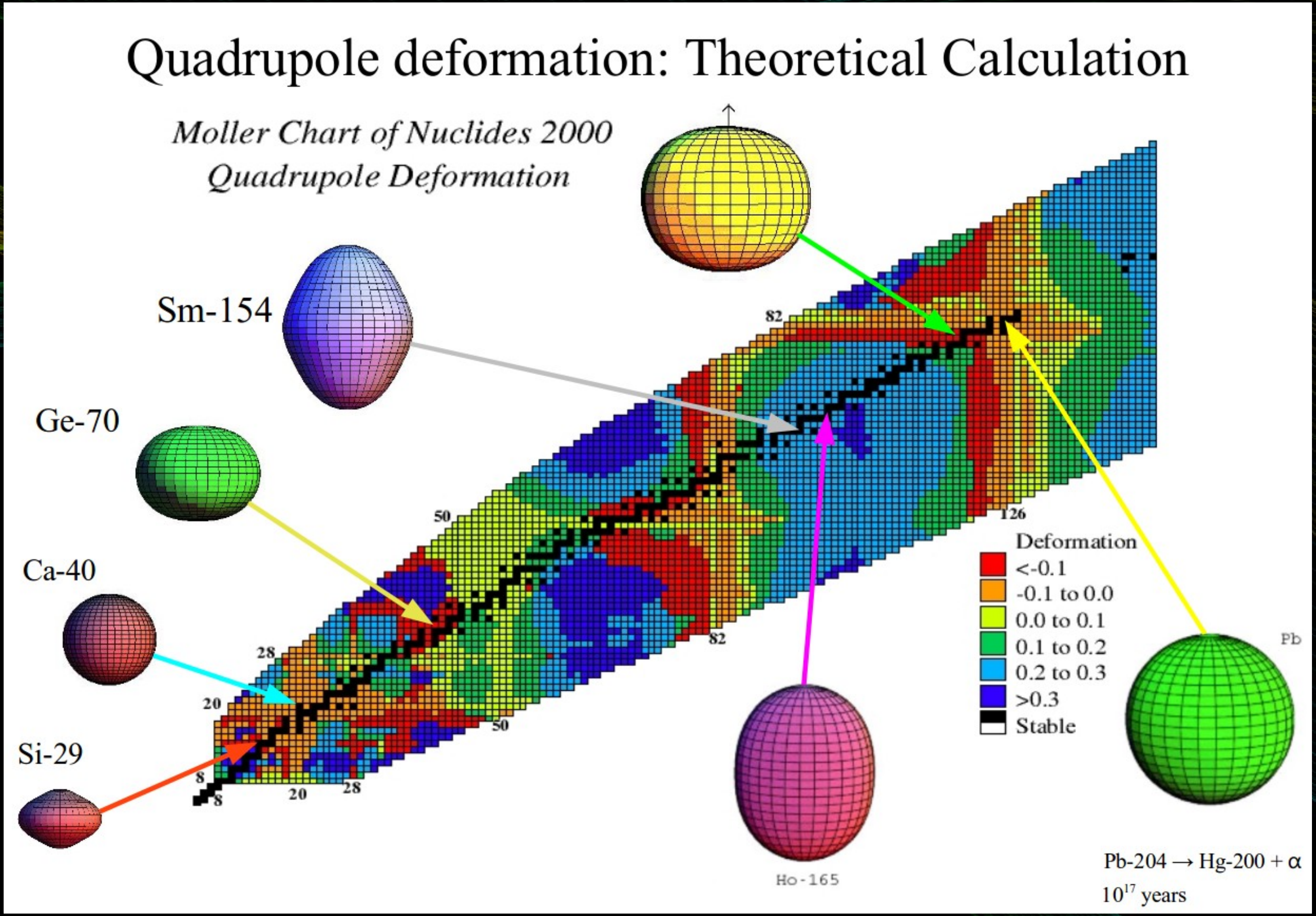


Deformed shell model

- Nilsson scheme: deformed oscillator potential
- Considering only quadrupole deformation



Deformations - Calculations

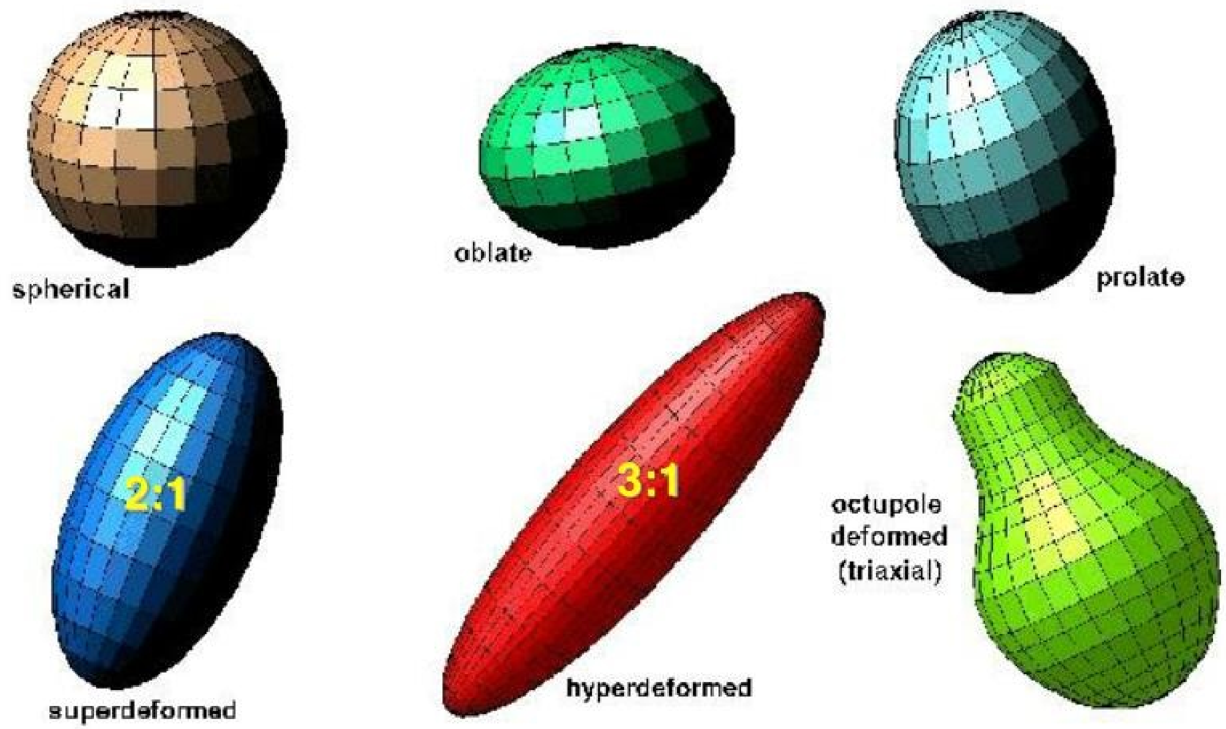


Deformations - Calculations

Nuclear Shapes

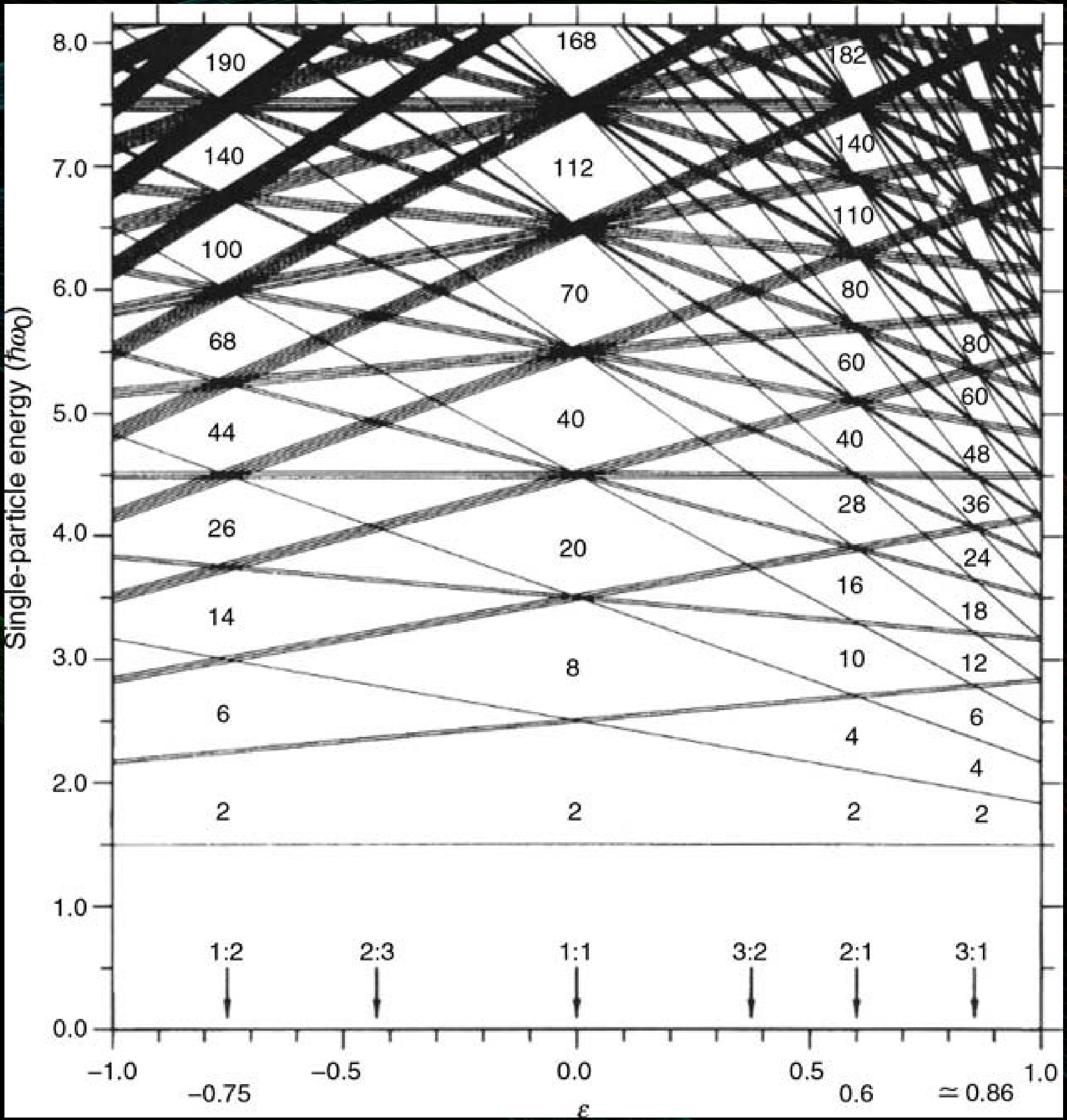
$$R(\theta,\varphi) = R_0(1 + \beta Y_{\lambda\mu}(\theta,\varphi))$$

$\lambda=2$; $\beta = 0$ spherical; $\beta < 0$ oblate (disk-like) ; $\beta > 0$ prolate (football-like)
$\lambda=3$; triaxial, octupole deformed



Extreme deformations

- New shell closures, new magic numbers at very large deformations
  - 2:1 axis ratio : **SUPERDEFORMATION**
  - 3:1 axis ratio: **HYPERDEFORMATION**
- Experimental technique:
  - gamma spectroscopy (SD)
  - fission resonances (HD)

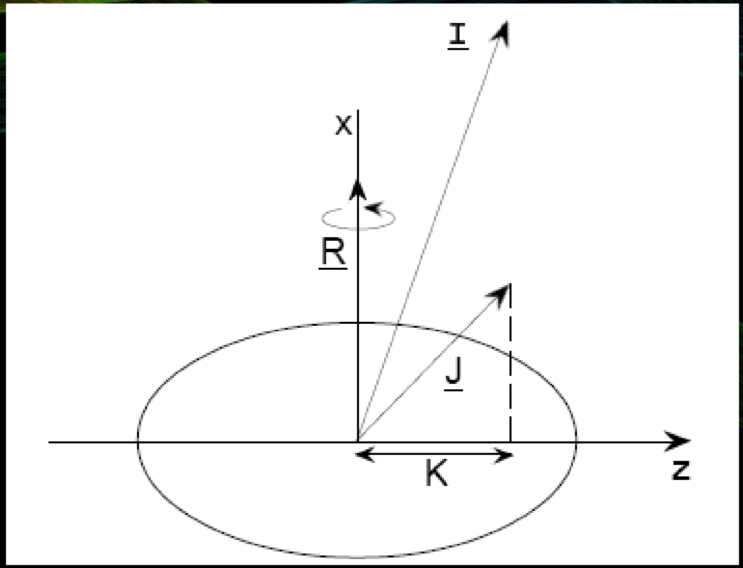


Nuclear rotations

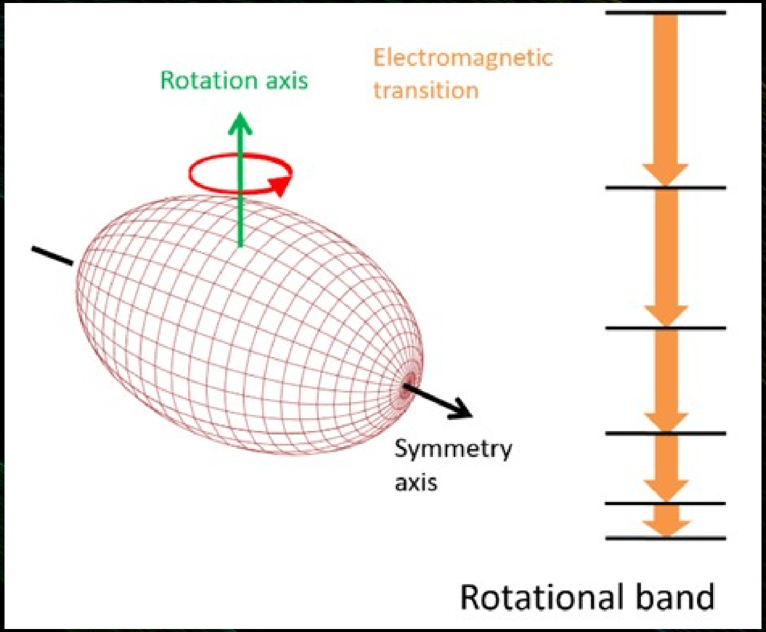
- Spherical nuclei cannot rotate according to quantum mechanics
- Deformed nuclei can rotate around the axis perpendicular to the nuclear symmetry axis

$$E_{rot}=\frac{\hbar^2}{2\Theta}[I(I+1)+J(J+1)-2K^2]$$

I=K,K+1,K+2,….



J is the total angular momentum of the valance nucleons
R is the angular momentum of the rotation around x
I is the spin of the nucleus



- For even-even nuclei and G.S. band: J=0 and K=0

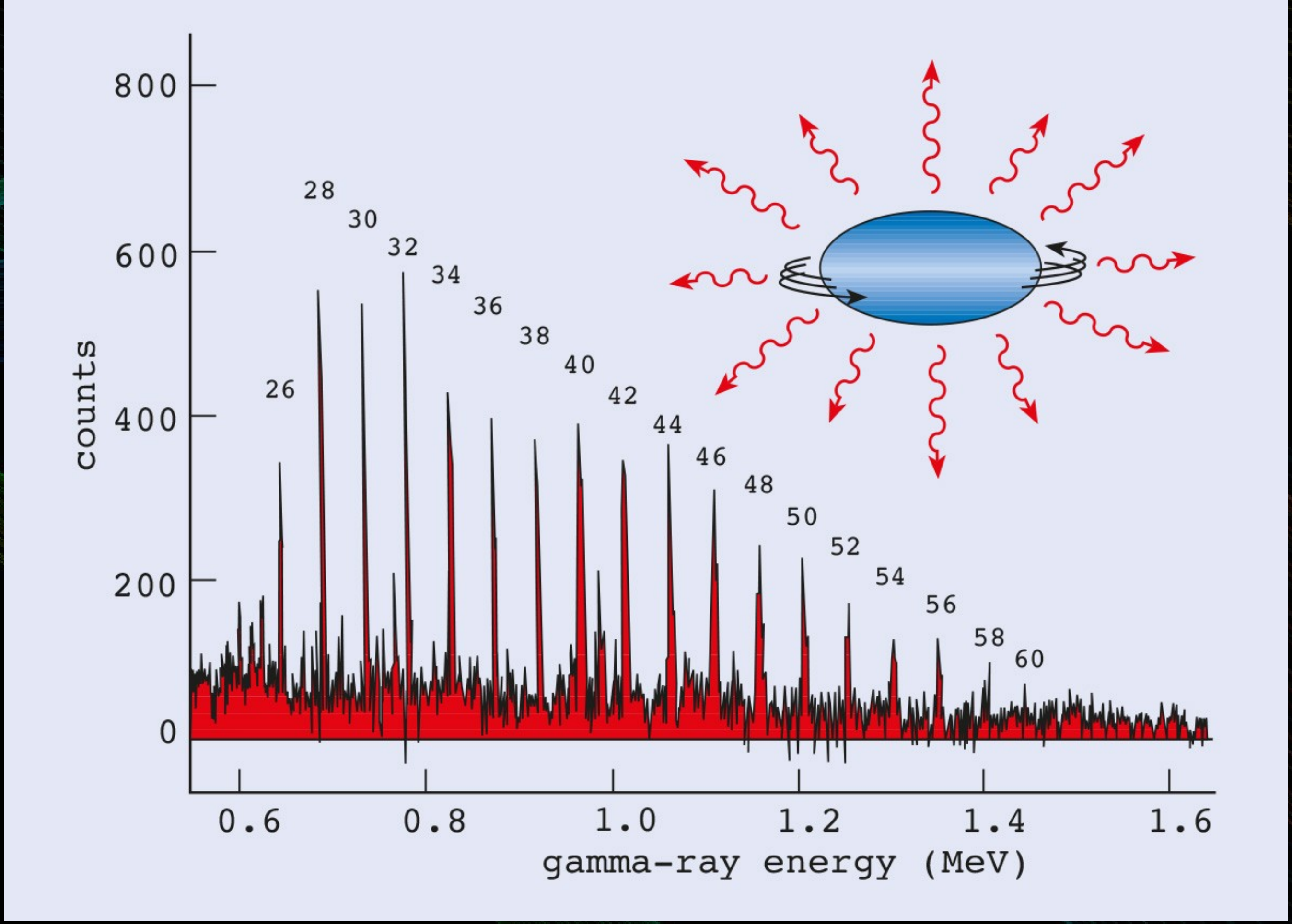
$$E_{rot}=\frac{\hbar^2}{2\Theta}I(I+1)$$

Nuclear rotations

- Moment of inertia, thus deformation can be determined experimentally from gamma energies
- Equal distances between gamma energies: fence spectrum

$$E_{rot}=\frac{\hbar^2}{2\Theta}I(I+1)$$

$$E_\gamma=E_x(I+2)-E_x(I)=\frac{\hbar^2}{2\Theta}(4I+6)$$



The unified nuclear model

- The nucleus can have rotations on top of vibrations
- The possible spins and parities of these rotational states is defined by the type of vibration