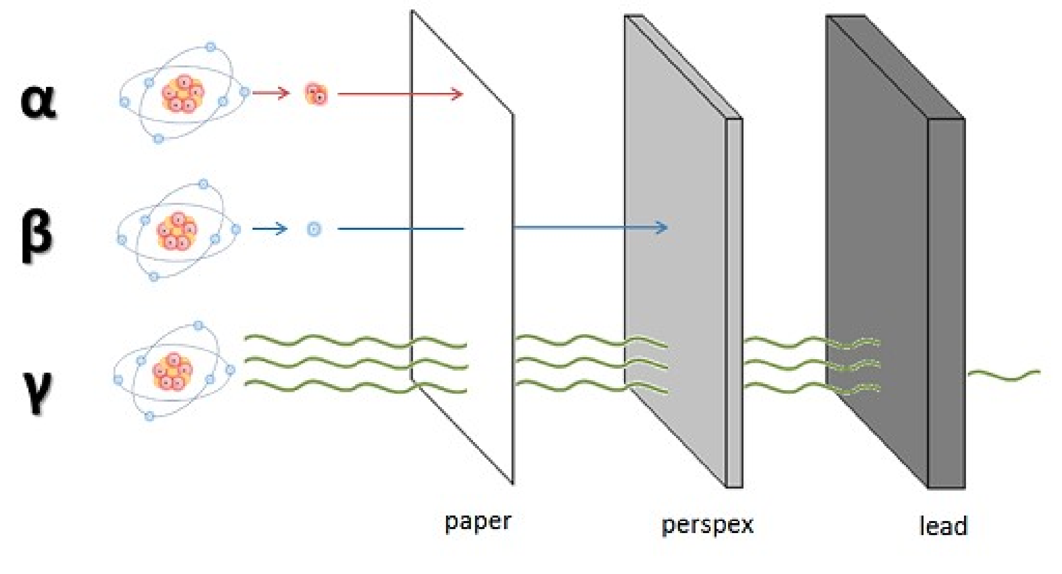
Új Nemzeti
Kiválóság Program

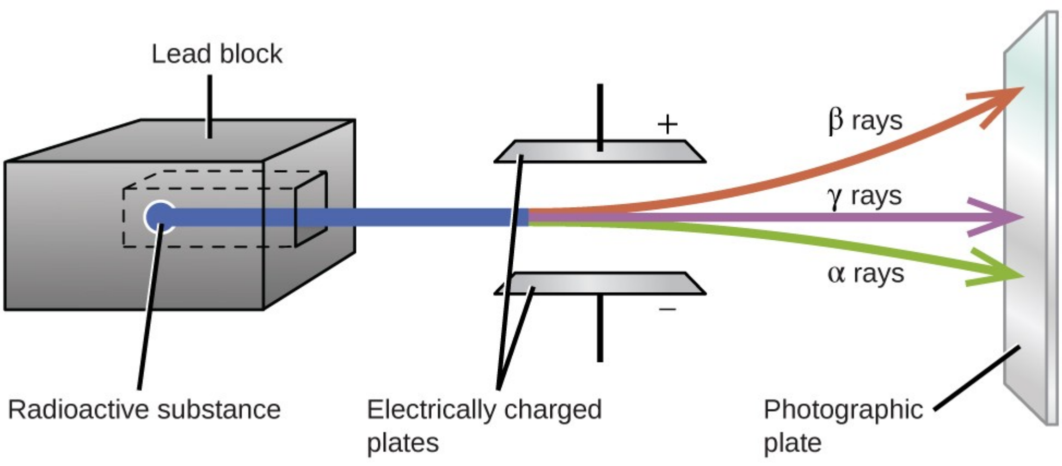
# IV. Radioactivity

**Lorant Csige**
Laboratory for Nuclear Physics
Hungarian Academy of Sciences

# Types of radioactivity

- α-radioactivity: $^{4}$He ions
  - short range (a paper absorbs)
  - high ionization power: dangerous in case of incorporation!
- β-radioactivity: e$^{-}$ or e$^{+}$
  - midrange → absorbed by an mm thick Al sheet (40-60 cm in air)
- γ-radioactivity: photon
  - energetic electromagnetic radiation from the excited nucleus
  - very long range, low ionization power



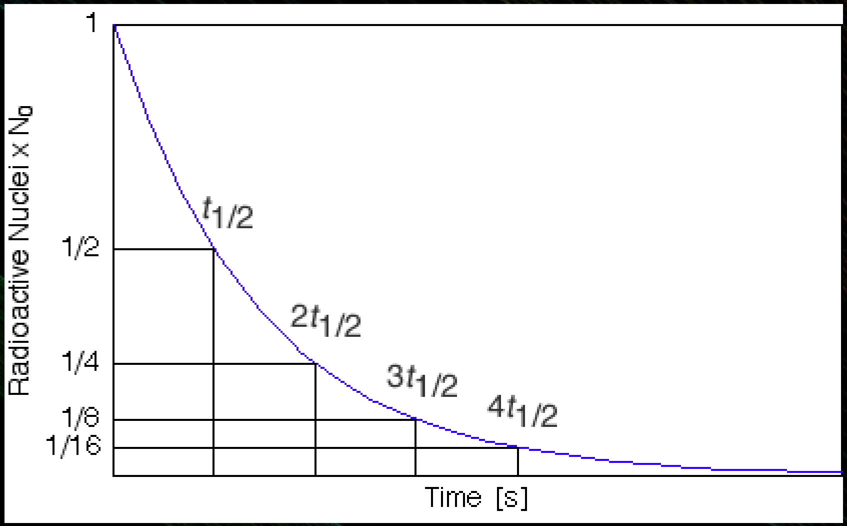


# Radioactivity: decay law

- A random (statistical) process with constant probability
- The number of decays proportional to the number of radiactive nuclei:

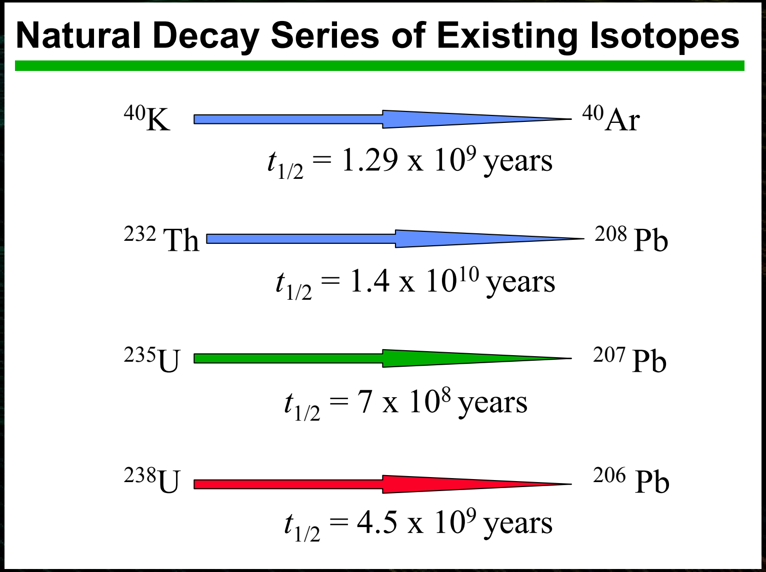
$$\frac{dN(t)}{dt}=-\lambda N(t) \Longrightarrow N(t)=N_0 e^{-\lambda t}$$

- λ is the decay constant: „speed” of the decay
- Mean lifetime (τ): 1/e of nuclei undergo decay → $\tau=1/\lambda$
  - level width: $\Gamma=\hbar/\tau$
- Half life ($T_{1/2}$): half of the nuclei undergo decay → $T_{1/2}=\ln 2/\lambda$
- Activity: $a(t)=-\lambda N(t)$
  - unit is Becquerel (Bq): 1 decay / second



# Nuclear decay chains

- Naturally occuring radioactive isotopes on Earth (and in our body even!)
- $^{40}$K is beta-radioactive while the rest is mostly alpha active
- The final products are Pb isotopes (Z=82, shell structure again...)
- Activity of different members within the decay chain can be calculated with differential equations:



**Natural Decay Series of Existing Isotopes**

$^{40}\mathrm{K} \rightarrow {}^{40}\mathrm{Ar}$, $t_{1/2} = 1.29 \times 10^9$ years

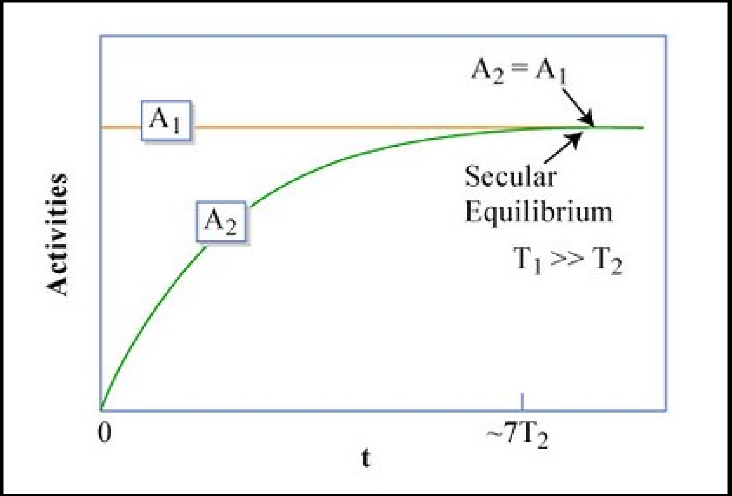
$^{232}\mathrm{Th} \rightarrow {}^{208}\mathrm{Pb}$, $t_{1/2} = 1.4 \times 10^{10}$ years

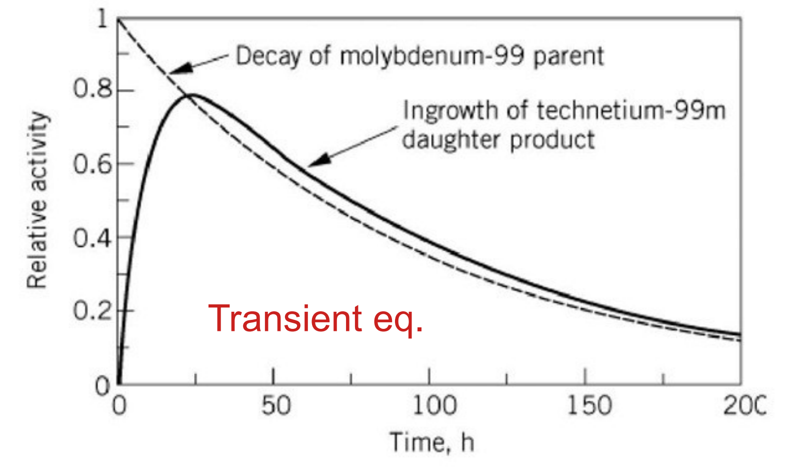
$^{235}\mathrm{U} \rightarrow {}^{207}\mathrm{Pb}$, $t_{1/2} = 7 \times 10^8$ years

$^{238}\mathrm{U} \rightarrow {}^{206}\mathrm{Pb}$, $t_{1/2} = 4.5 \times 10^9$ years

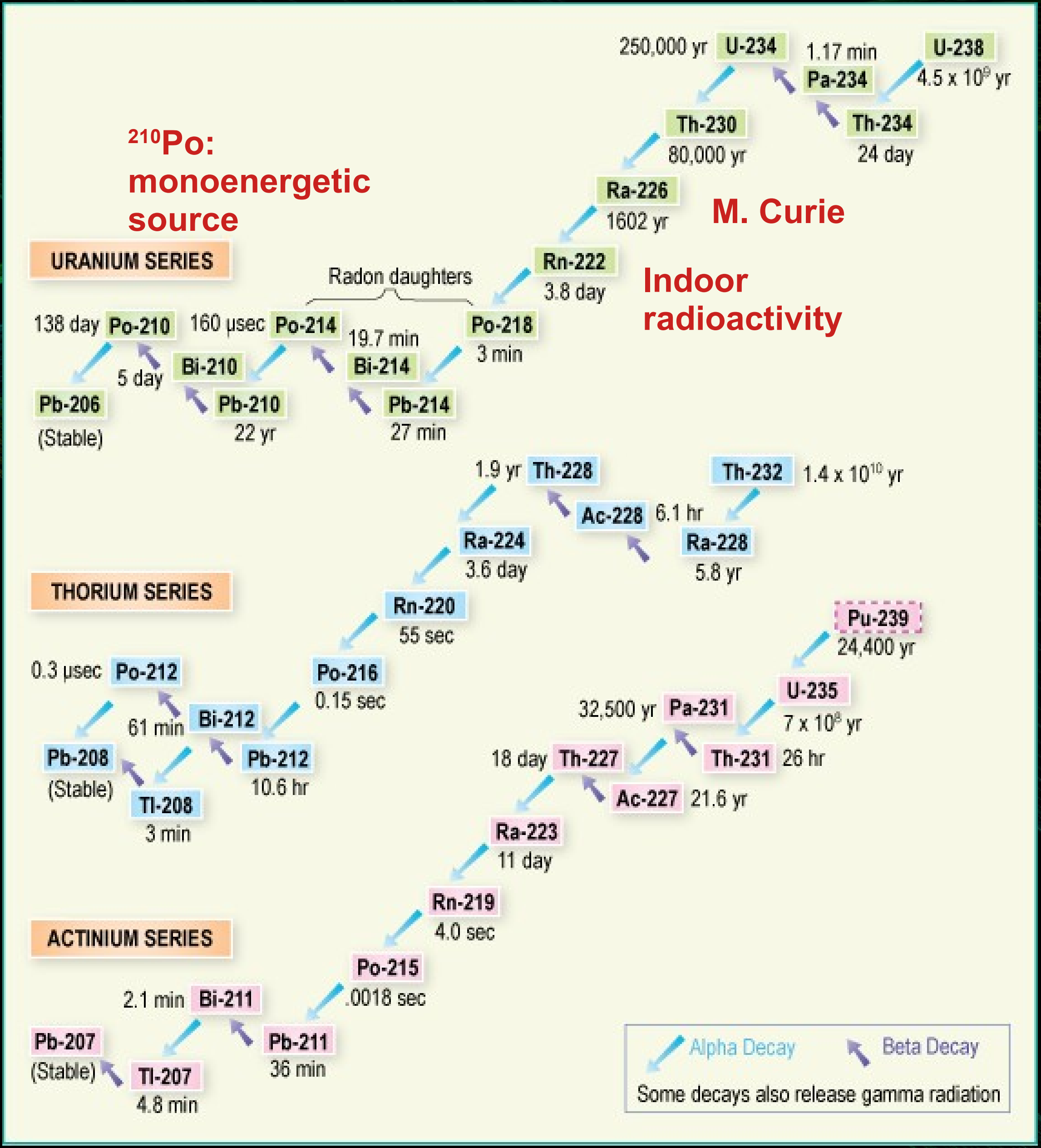
$$\frac{dN_1(t)}{dt} = -\lambda N_1(t)$$

$$\frac{dN_2(t)}{dt} = \lambda_1 N_1(t) - \lambda_2 N_2(t)$$





# Nuclear decay chains



**$^{210}$Po: monoenergetic source**

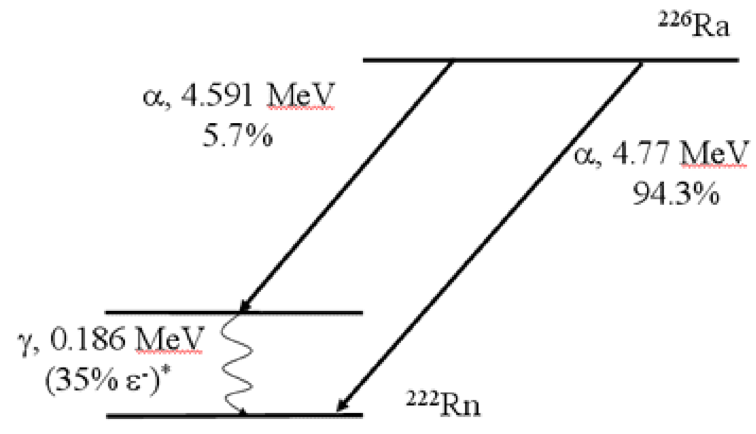
**M. Curie**

**Indoor radioactivity**

# Alpha decay

$$^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}Y_{N-2} + {}^{4}_{2}He_{2}$$

- α-particle is a $^4$He nucleus ($^4He^{++}$ ion)
- the energy balance of alpha decay and the condition for alpha decay: **$Q=M_X(A,Z)-[M_Y(A-4,Z-2)+M_{He}(4,2)]>0$** (from LDM → Z>73)

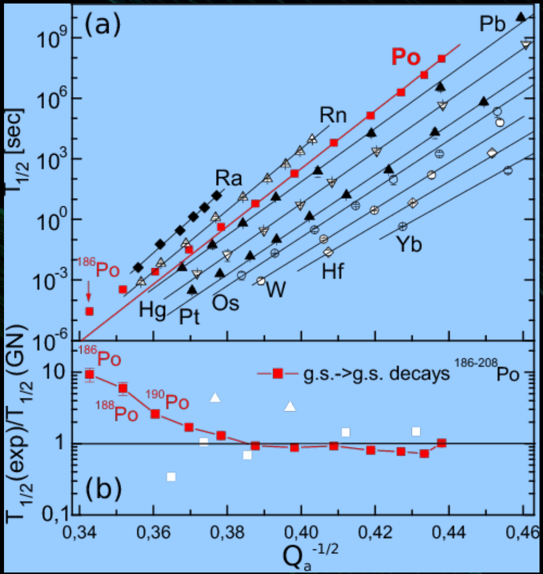


- α-decay of $^{226}$Ra
- α-particle energy is determined by the initial and final states
- transitions to higher (excited) levels are possible (mostly to low lying rotational states of deformed daughter nuclei) but with quickly decreasing probability

- 4 MeV < $E_\alpha$ < 9 MeV while $10^{-7}$ s < $T_{1/2}$ < $10^{10}$ year (!!)
- relation between $E_\alpha$ and $T_{1/2}$ within the 3 natural decay chains is the Geiger-Nuttall rule (1911)

$$lg\lambda = a \cdot lgE_\alpha + b$$

where a is the same for all chains, b is not



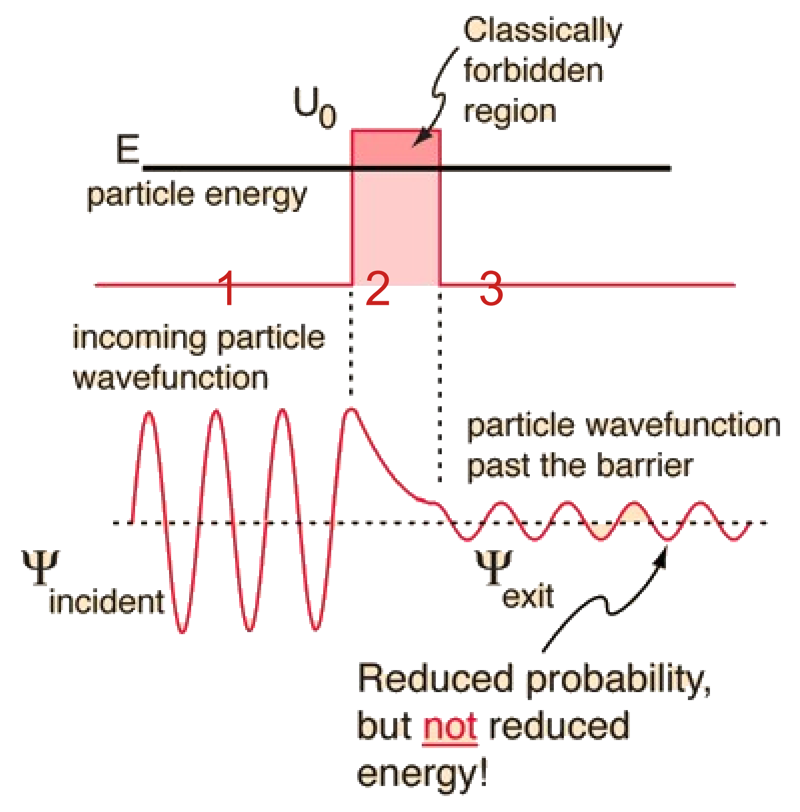
# Alpha decay – quantum tunnelling

- *Gamow*: understanding the Geiger-Nuttall rule within quantummechanics
- From the solving the Schrödinger equations in the 3 different region (where $U_0 < E$ [1-3] and $U_0 > E$ [2])

$$\Delta\Psi + (2m/\hbar^2)(E - V)\Psi = 0$$

$$D = \exp\left[-\frac{2}{\hbar}\sqrt{2m(U_0 - E)}\,x_0\right]$$

transition probibility



- If $U_0 = Z_{nucleus} z_\alpha e^2/r$ is the Coulomb potential (3 dimensional) and $E_\alpha$ is the energy of the α-particle → GN rule can be understood

- If $l_\alpha > 0$ (if it is allowed, see next)→ probability is decreased by the centrifugal barrier:

$$V_{cf}(r) \approx \frac{l(l+1)\hbar^2}{3mr^2}$$

# Alpha decay

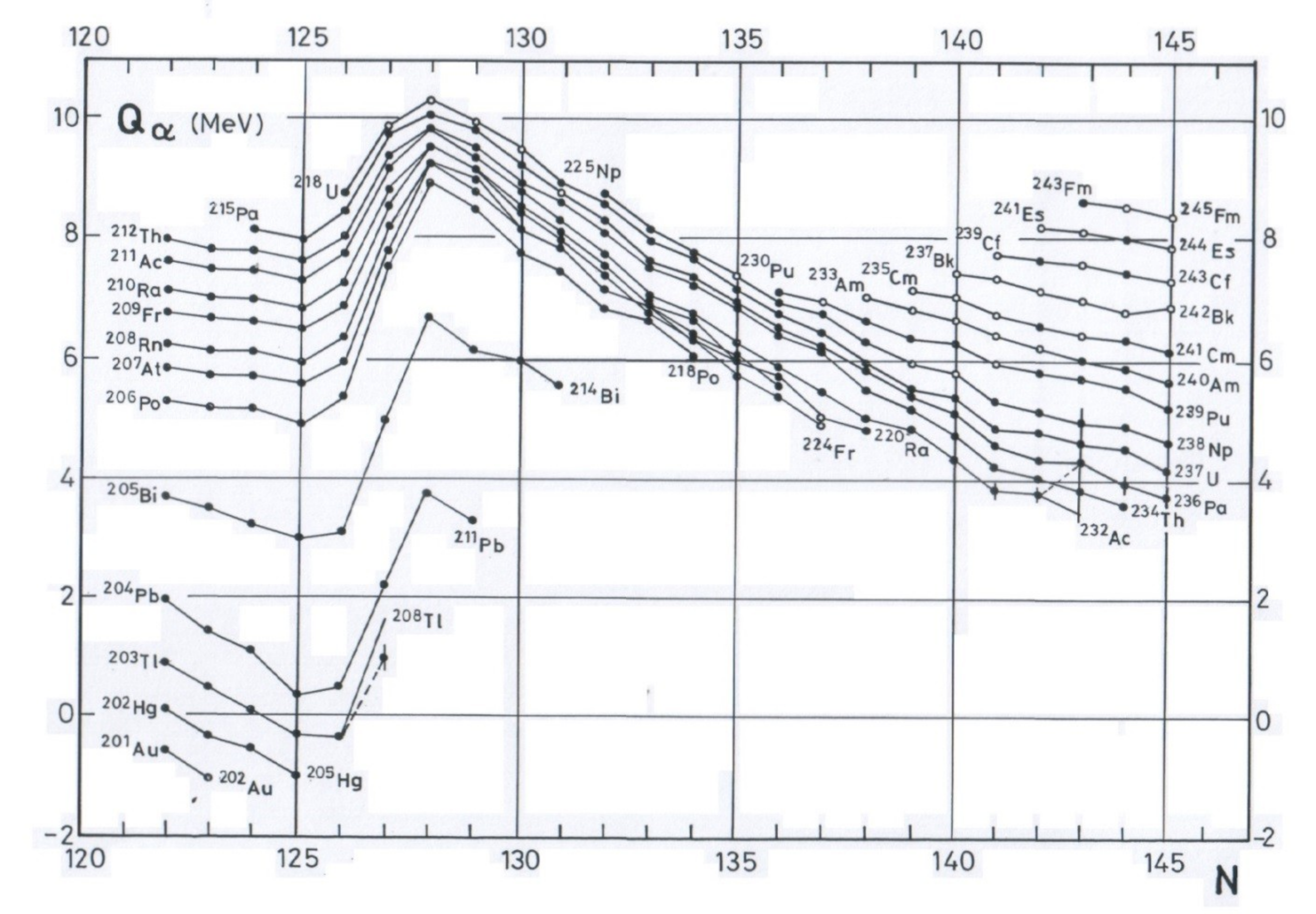
- Controlled by the strong force
- Kinematics: momentum conservation
  - $|p_\alpha|=|p_{daughter}| \rightarrow E_\alpha=Q_\alpha[M_{daughter}/(M_{daughter}+M_\alpha)]$ ($\rightarrow E_{daughter}$ is typically small: ~2%)
- ***P*** parity and ***T*** isospin is conserved → selection rules
  - $\boldsymbol{T}_\alpha = 0 \rightarrow$ parent and daughter nuclei have the same isospin
  - $\boldsymbol{P}_\alpha=+1$ and $\boldsymbol{I}_\alpha=0 \rightarrow |I_p-I_d| \leq l_\alpha \leq |I_p+I_d|$ and $P_p=(-1)^{l_\alpha}P_d$

- Some remarks:
  - From D to λ : ***λ=PνD*** (P: probability of an α formation within the nucleus and ν: the frequency of the α particle „hitting” the surface) →

$$lg\,\lambda=\frac{A}{\sqrt{E_\alpha}}+B$$

  - A forbidden alpha transition is not „absolutely” forbidden (but decreased in probability by a factor of $10^7$-$10^{14}$)
  - Centrifugal barrier is not so large → only little dependence on $l_\alpha$

# Alpha decay systematics



- Large Q at N=128 → shell structure

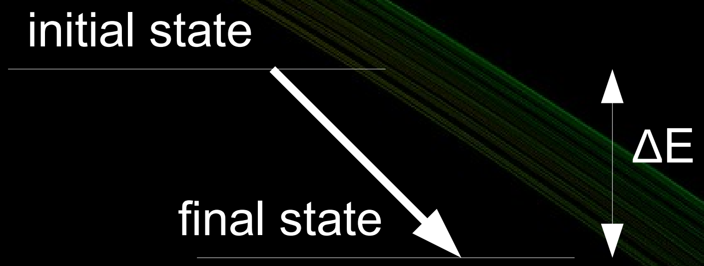
# Beta decay

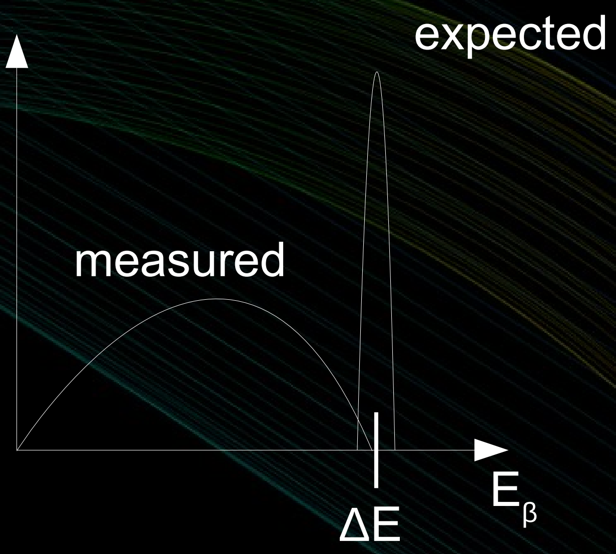
- Transmutation of an unstable nucleus to an isobar one with $\Delta Z = \pm 1$
- Condition: $E_{\beta-} = [M_{at}(A,Z) - M_{at}(A,Z+1)]c^2$ and $E_{\beta+} = [M_{at}(A,Z) - M_{at}(A,Z-1) - 2m_e]c^2$
- Experimental observations: $0.02 < E_\beta < 16$ MeV and $10^{-2}$ s $< T_{1/2} < 10^{15}$ years

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+1}Y_{N-1} + e $$

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z-1}Y_{N+1} + p $$

$$ {}^{A}_{Z}X_{N} + e \rightarrow {}^{A}_{Z-1}Y_{N+1} $$





- Experimental tecnique: magnetic spectrometers
- Instead of $E_\beta = \Delta E$ a continuous $E_\beta$ is measured with $0 < E_\beta < \Delta E$!
- maximum yield is at $E_{max}/3$ (for natural radioactivity $E_\beta = 0.25$-$0.45$ MeV)
- more symmetric β-spectrum for light nuclei: $E_{max}/2$

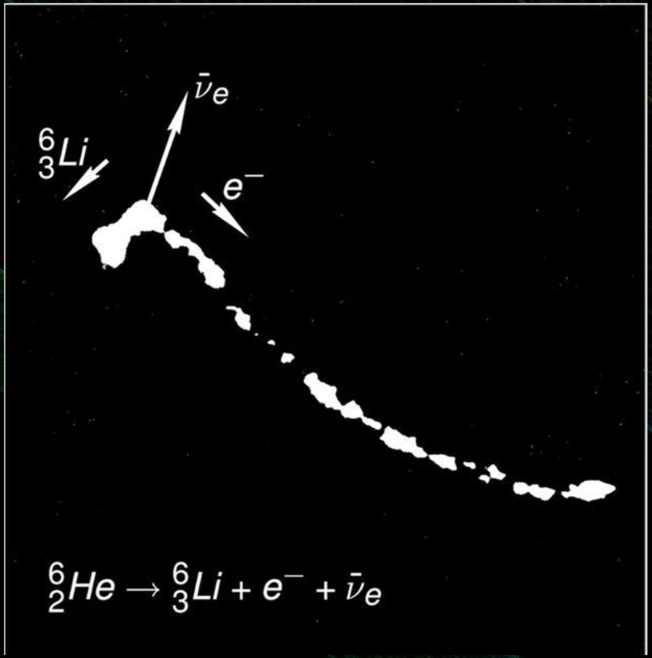
# Beta decay

- Explaining the continuous $E_\beta$ spectrum:
  - monochromatic $e^-$ are emitted, but scattering on the electron „clouds” of atoms
  - no energy conservation in β decay....
  - W. Pauli (1931): a third particle takes away the missing energy!
    - the particle should have zero charge (charge conservation), small mass and $s=1/2\hbar$

$$n \rightarrow p + e + \tilde{\nu}_e$$

**neutrino hypoteses**

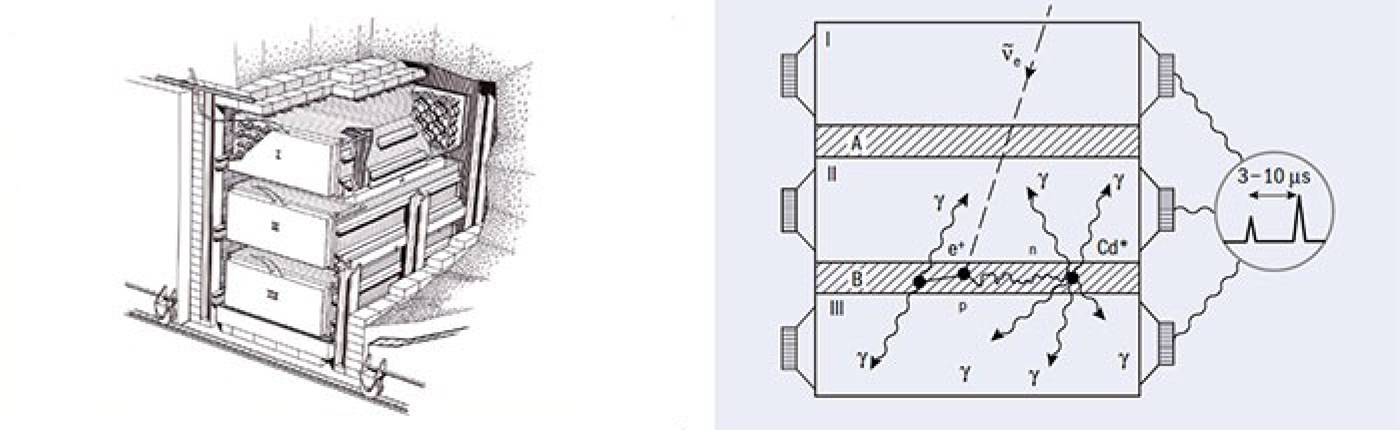
- Szalay – Csikai experiment (1956):
  - indirect observation of neutrino
  - momentum conservation should be statisfied
  - photon of a decay in a Wilson cloud chamber in Debrecen
  - (unfortunatelly published only in 1957)



# Beta decay

- *Reines and Cowan* (1956): inverz beta decay of neutrons

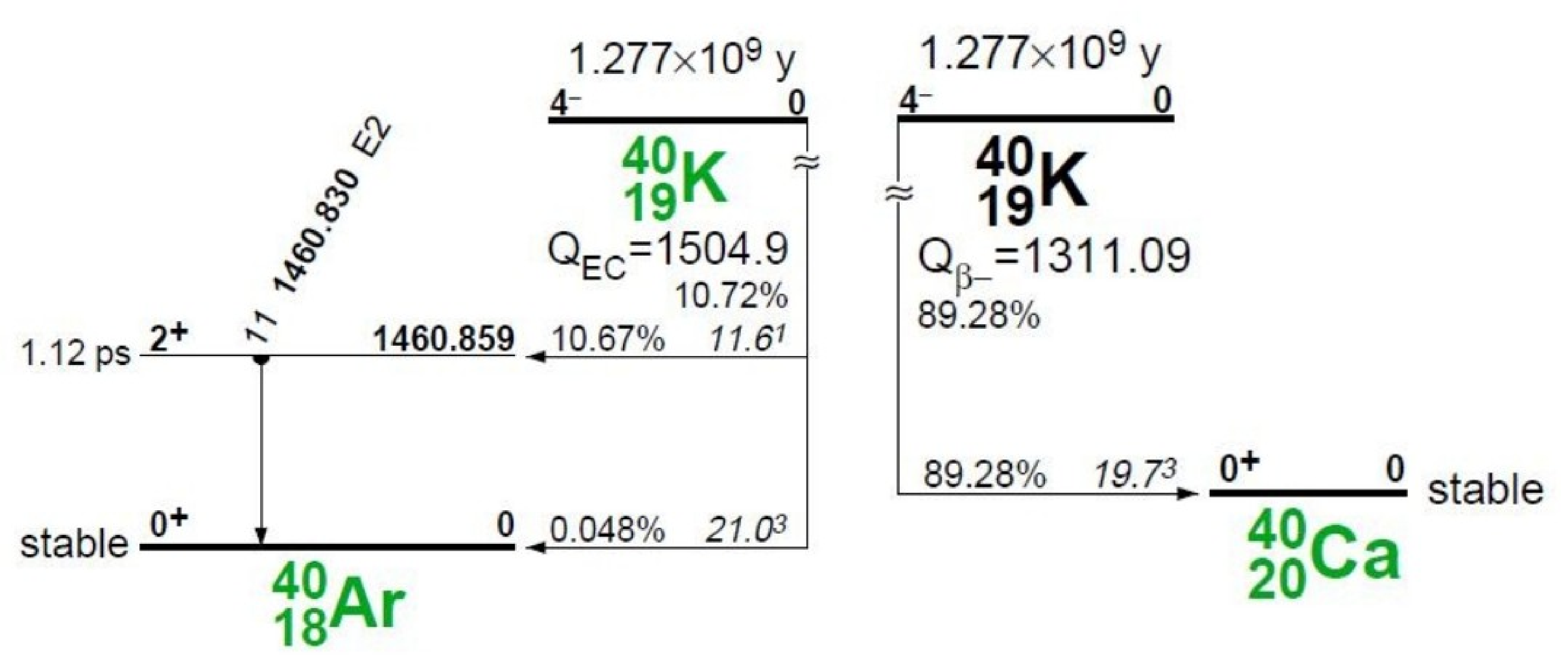
$$\tilde{\nu}_e + proton \rightarrow n + positron$$



- *A, B* volumes: $CdCl_2$ water-solution → Cd neutronabsorber, water moderator
- *I* and *II* and *III* volumes: liquid scintillator material (1 $m^3$)
- Triple coincidences of
  - the positron annihilation (2x511 keV γ photon)
  - neutrons, after moderation, are absorbed by Cd which gives delayed γ photons
- Cross section was determined to be $\sigma=10^{-43}$ $cm^2$ → $10^{17}$ km mean free path in condensed material...

# Beta decay: an example

- decay scheme of $^{40}$K: both type of beta decay!
- transitions to higher levels → gamma decay



# Beta decay

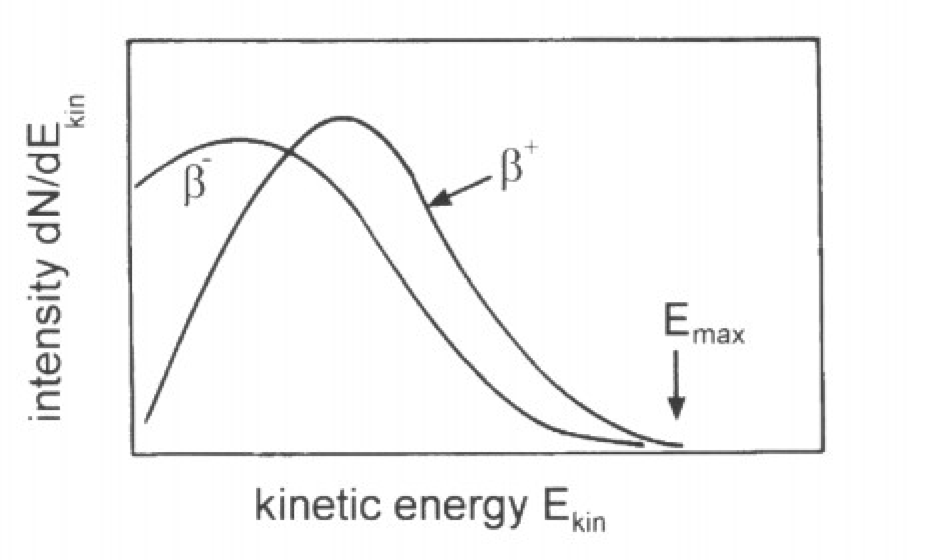
- Fermi (1934): understanding the beta spectrum
  - beta decay lifetime is typically long → the interaction is weak → perturbation theory can give the transition probability
  - electron and neutrino can be in all the quantum states with equal probability
  - the probability of emitting an electron with E - E+dE energy:

$$P(E)\,dE = 2\frac{\pi}{\hbar}|H_{i,f}|^2 \rho(E)\,dE$$

$\rho(E)$: final state density
$H_{i,f}$: transition matrices (depend on the initial, the final state and the interaction)

$$\rho(E) = \sqrt{E^2 - m^2c^4}\sqrt{(E_0 - E)^2 - m_\nu^2 c^4}\,E\,(E_0 - E)$$

(if $m_\nu=0$ than simpler)

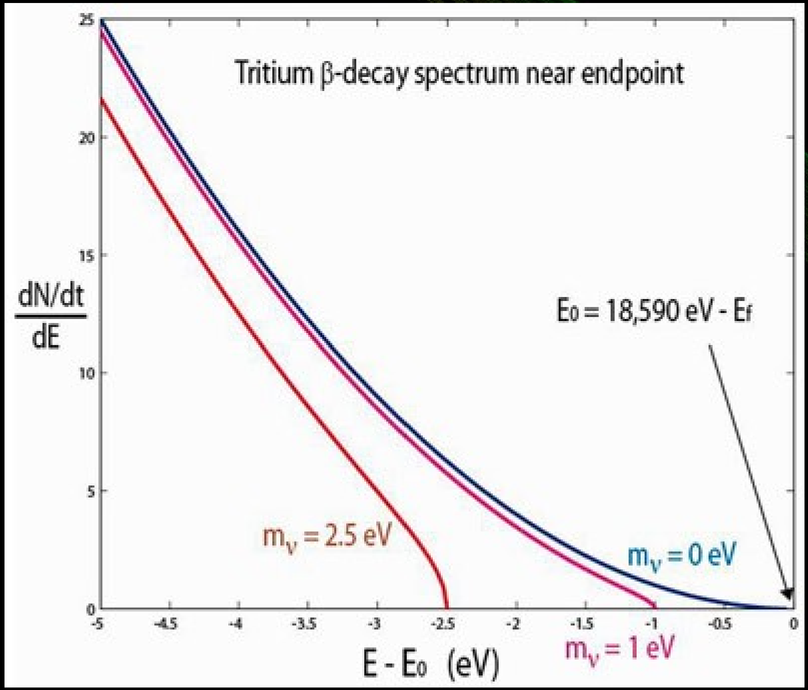


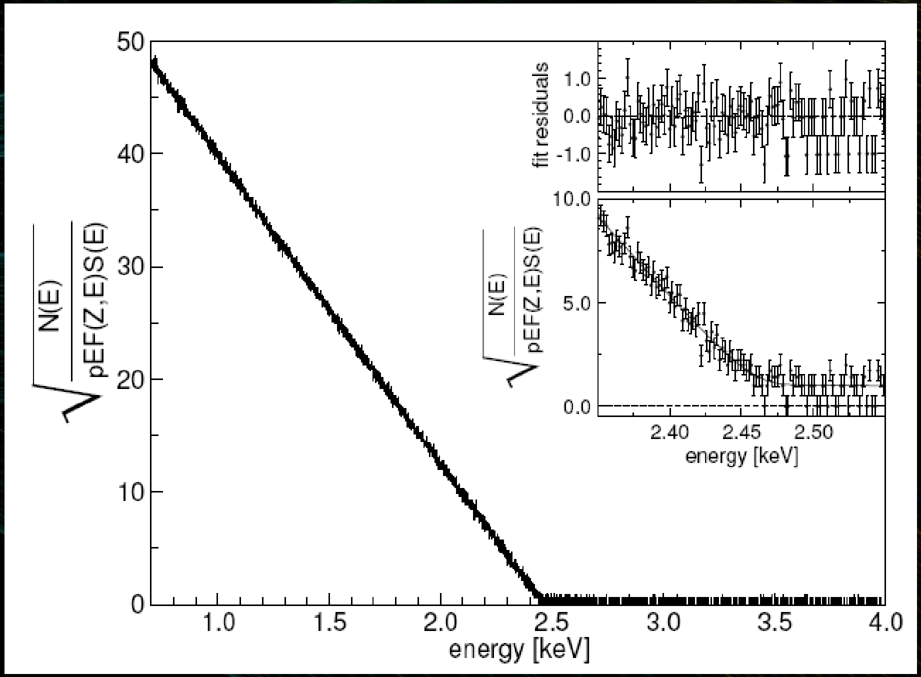
- Spectrum starts at mc² (E is the total energy above)
- Coulomb correction: nucleus is positively charged → attracts electrons, pushes positrons → spectrum is shifted / distorted
- Fermi introduced a factor: F(Z,E)

# Beta decay theory

- Fermi-Kurie plot: for convenicence it is better the plot a normalized form and not the spectrum N(E)

$$\sqrt{\frac{N(E)}{E\sqrt{E^2-m^2c^4}\,F(Z,E)}}$$





- for allowed beta decay, it is linear
- $E_0$ is defined by the crossing with the X-axis $E_e$
- if neutrino has mass, than the electron energy end-point differs from $E_0$ by $m_\nu c^2$
- see the scale!!

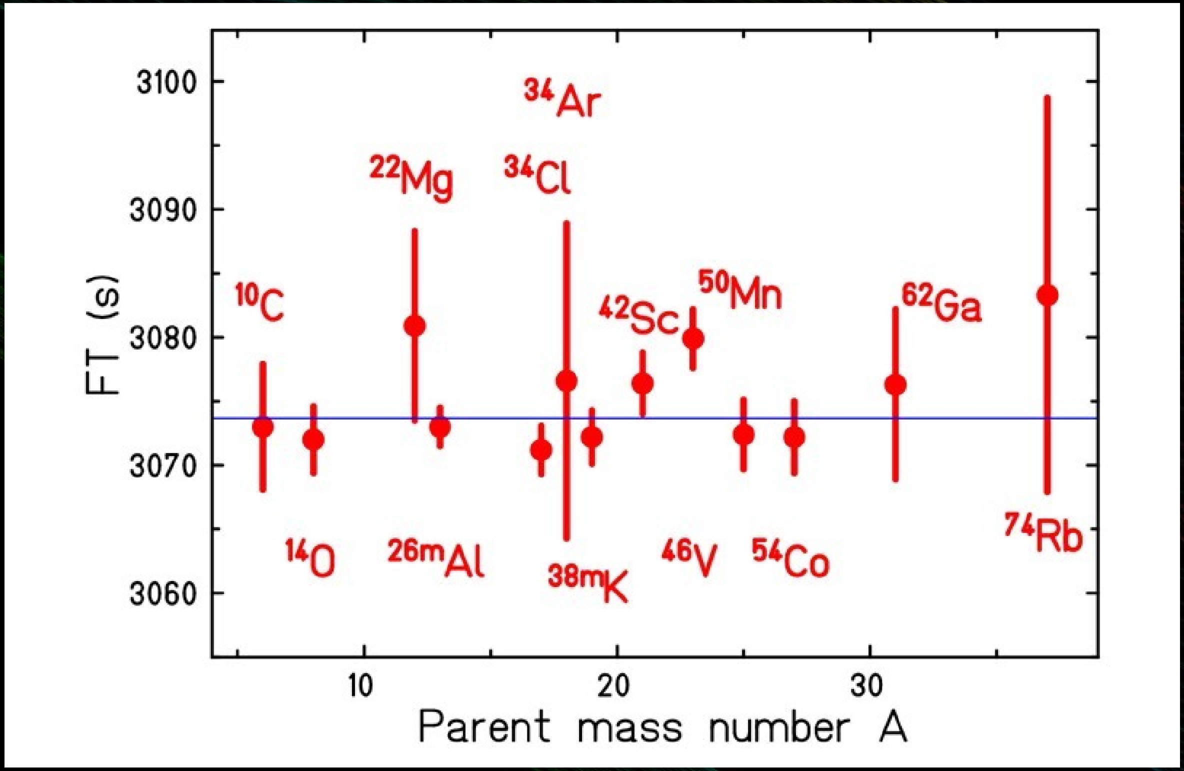
# Beta decay: theory

- Total probability of beta decay:

$$\lambda=\frac{1}{\tau}=f(Z,E_0)=\int_0^{E_0}\rho(E)F(Z,E)\,dE\approx\int_0^{E_0}E^4\,dE=E_0^5 \quad \Rightarrow \quad f\cdot\tau\approx constant$$

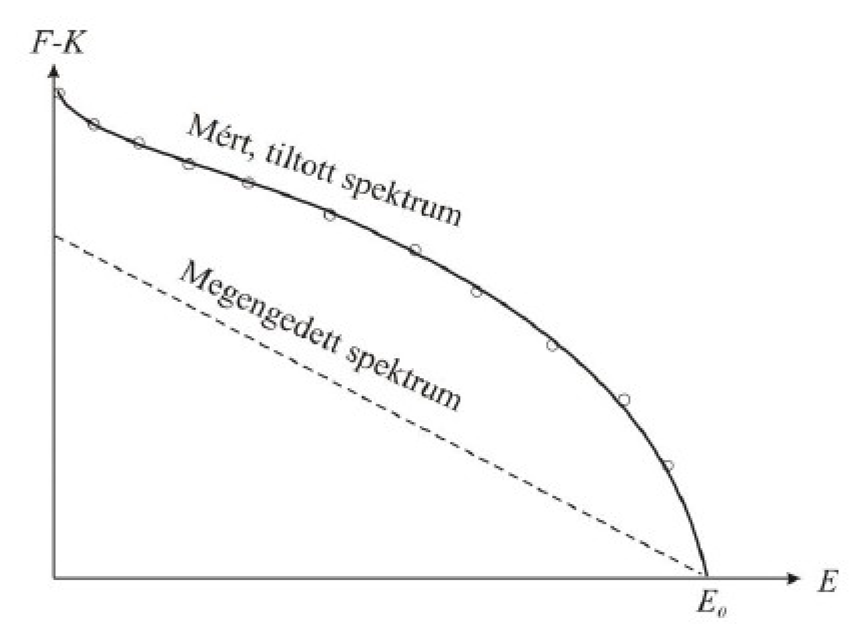
- $f(E_0)\tau \rightarrow$ comparative half life is constant for superallowed, allowed, and forbidden transitions

| transition type | log(ft) |
|---|---|
| superallowed | 2.9-3.7 |
| allowed | 4.4-6.0 |
| first forbidden | 6.0-10 |
| second forbidden | 10-13 |



# Beta decay: theory

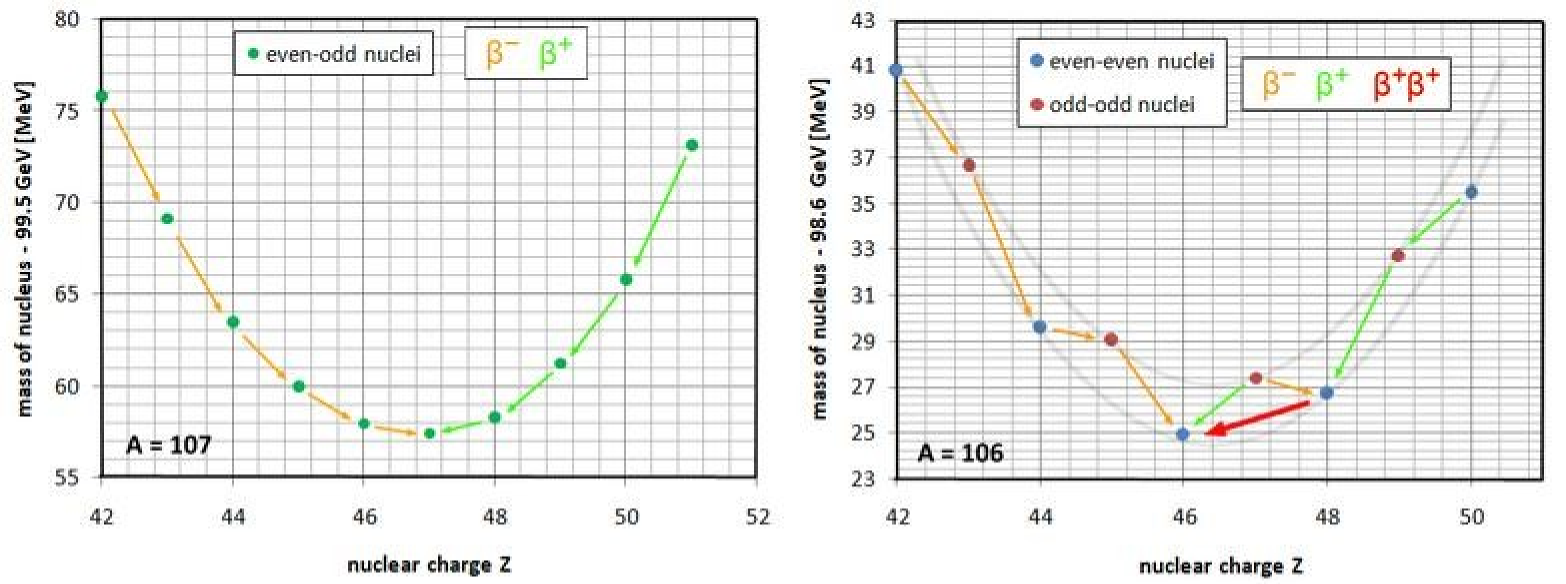
- Allowed transitions: $l=0$ and $P_1=P_2$
  - Fermi transition: $s_e+s_\nu = 0$ (spins are antiparallel)
  - Gamow-Teller transition: $s_e+s_\nu = 1$ (spins are parallel)
    - Sun: $2p \rightarrow d+e^+ + \nu$
    - Pauli principle: s(2p)=0 but s(d)=1 !!
    - Fermi transition is not possible.

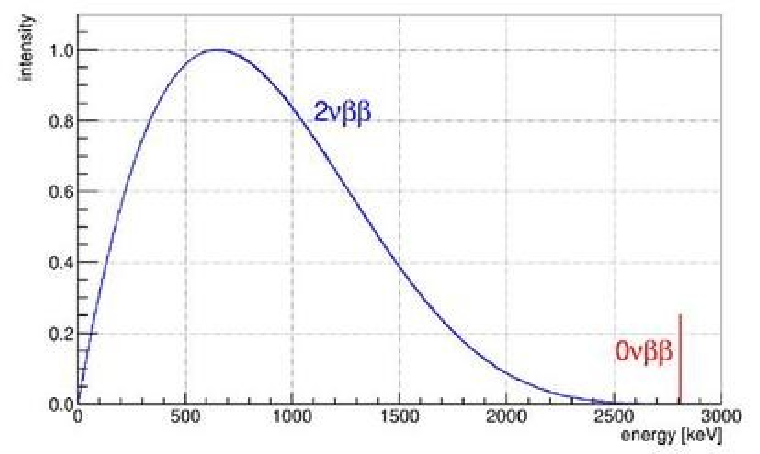


- Forbidden transitions: $l>0$ and/or $P_1 \neq P_2$
  - transition probability is very small
  - Kurie plot is deviated from linear
  - charaterized numerically by the lg(Ft) value

# Double beta decay

- Sometimes energetically beta decay is not allowed between even-even isobars → but it is for Z+2!! → double beta decay is allowed





(if $\bar{\nu}\neq\nu$) 2νββ: $2n \rightarrow 2p+2e^-+2\bar{\nu}$

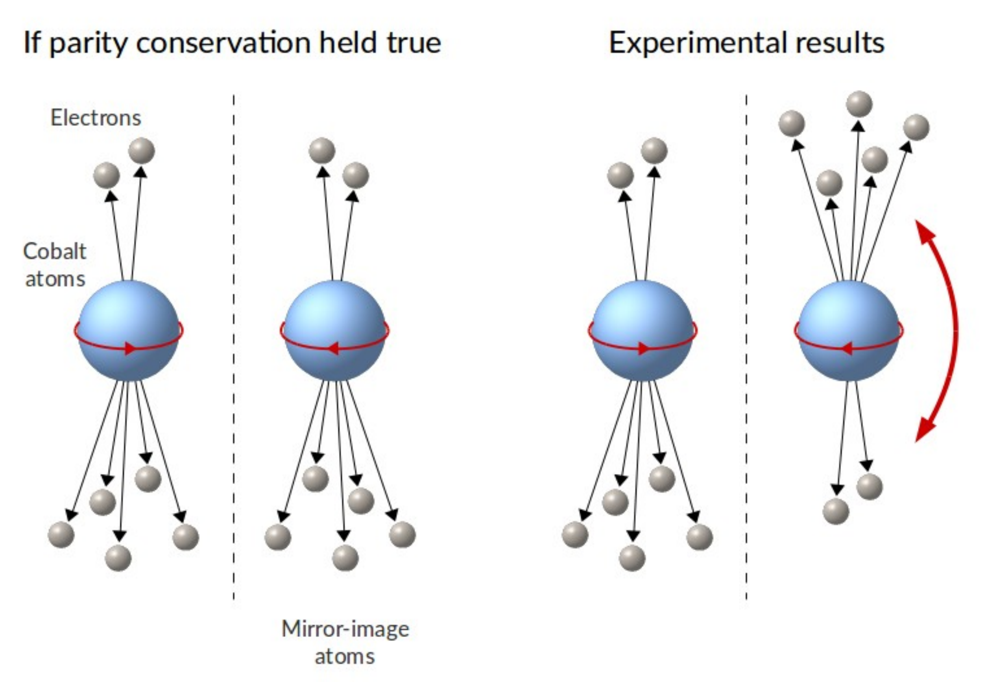
(if $\bar{\nu}=\nu$) 0νββ: $n \rightarrow p+e^-+\underline{\bar{\nu}\ \ +n} \rightarrow p+e^-$

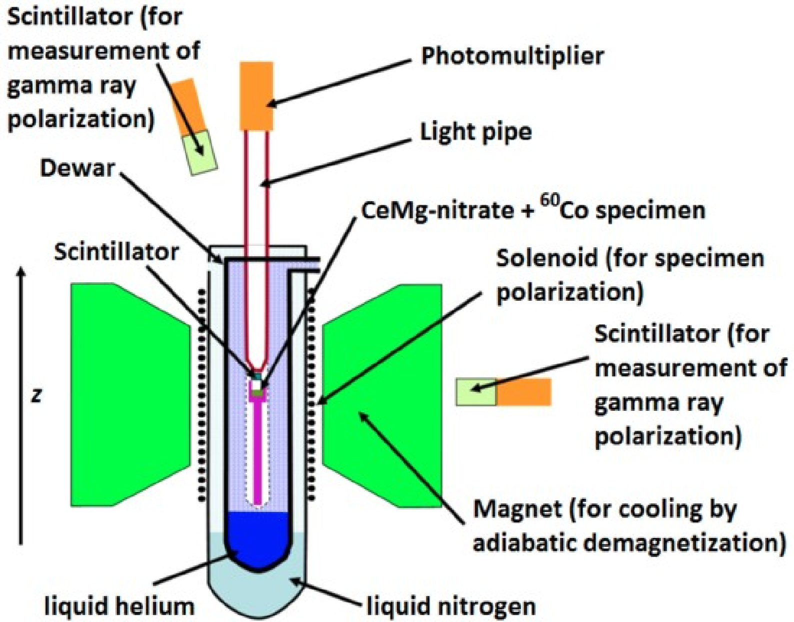
- N(E) gives the transition energy E0
- Neutrino is a majorana particle? Very intense research field today!

# Beta decay: parity violation

- C. Wu experiment (1956):
  - in strong and EM interaction parity is conserved
  - what about weak interaction, in beta decay?

$$^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni} + e^- + \bar{\nu} + 2\gamma$$

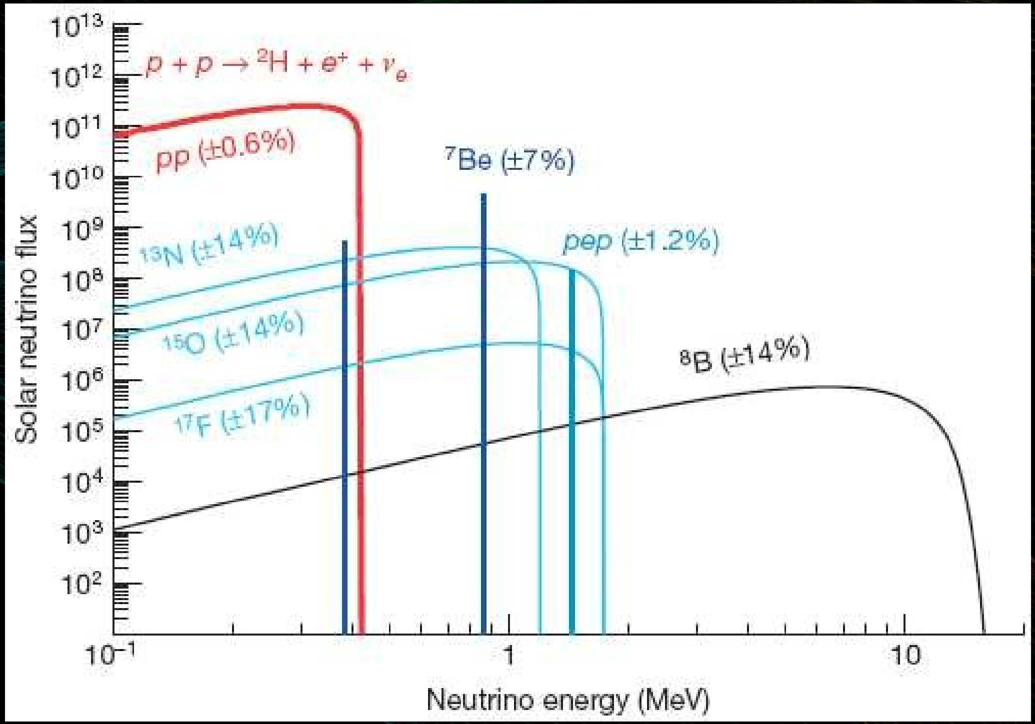


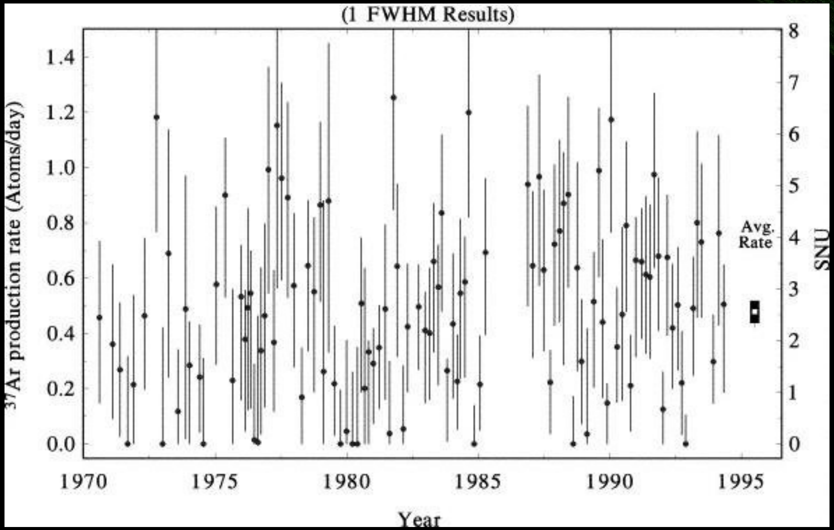


- Top science, top technology!
  - needed high magnetic field (10T), and very low temperature to align $^{60}$Co spin
  - T=0.003K (!!) was achieved by adiabatic cooling
- Result: $e^-$ emission preferred the direction which is opposite to the $^{60}$Co spin!!

# Neutrino physics

- R. Davis (1969): measured the solar neutrino flux
- Sun is an extreme source of neutrinos: 64 billions of neutrinos per second per 1cm$^2$ !!
  - But very low cross section: $10^{17}$ m
- pp-cycle: 98.5% of power (rest is CNO cycle)
  - allmost all neutrinos are low energy E<0.42 MeV
  - but $^8$B and $^7$Be neutrinos are energetic: E<14 MeV





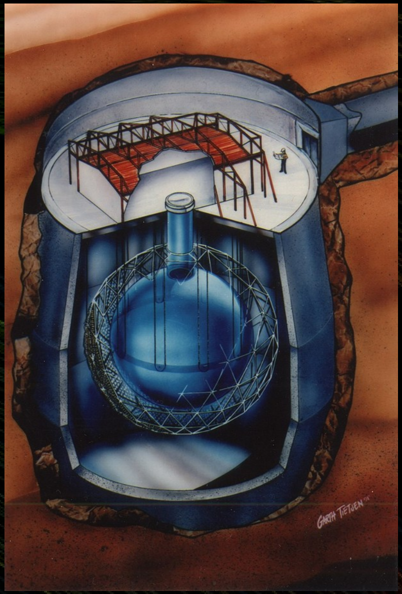
- The experiment was at Homestake mine with 615 tons of tetrachlore-etan, $C_2Cl_4$ a cleaning fluid

$$^{37}Cl + \nu_e \rightarrow \ ^{37}Ar + e^- \quad E>0.814\ MeV$$

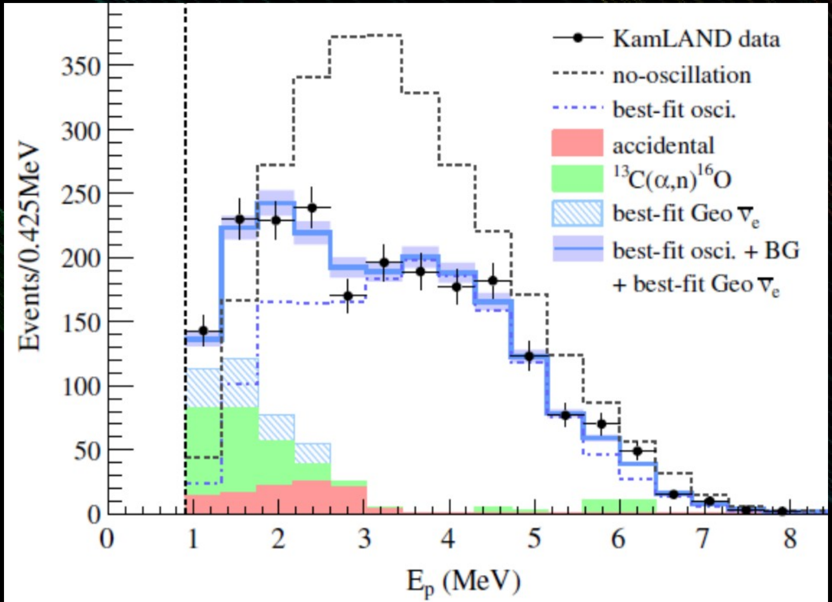
- $^{37}$Ar decay was detected
- Measured 0.48 atoms/day instead of 1.5 atoms/day

# Neutrino physics

- Further experiments:
  - Solar and atmospheric: Super-Kamiokande, Japan, 1998
  - Solar neutrinos: Sudbury Neutrino Observatory, Canada, 2001
    - sensitive to all type of neutrinos

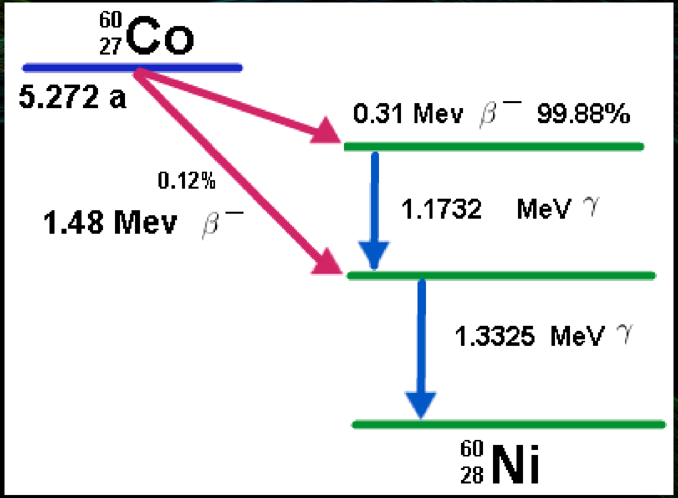


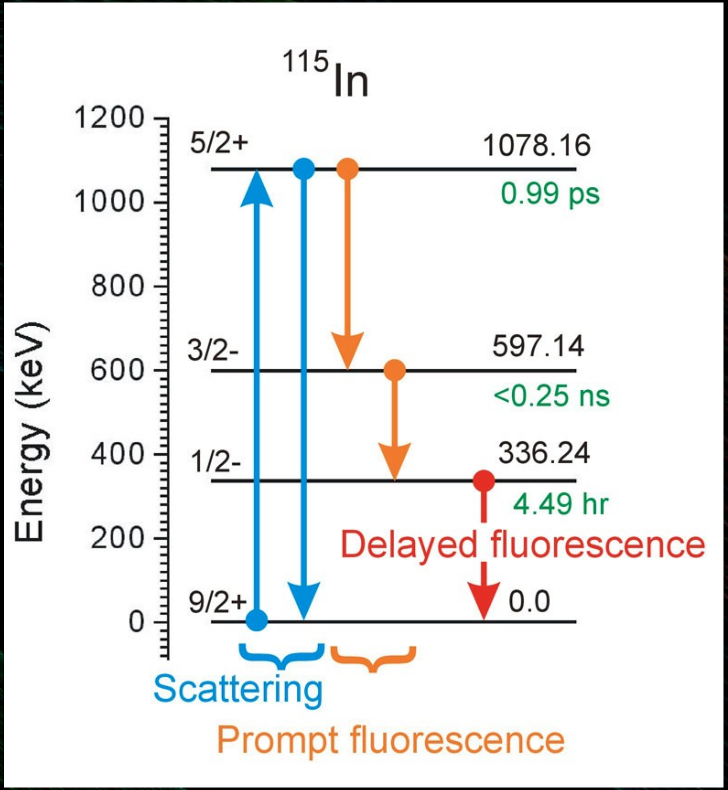
- Solution: neutrino oscillations
  - electron, muon and tau neutrinos can transform to each other
  - detectors are sensitive only to electron neutrinos → the rest is „missing”
  - But! Oscillation only can take place if at least one of the neutrinos has mass!
- KamLAND experiment, 2005-2008



# γ decay

- Excited states of a nucleus → de-excitation by γ photon emission
- Electromagnetic radiation with short wavelength ($10^{-8} > \lambda > 10^{-11}$ cm)
- Excitation is due to a) α (typically very low lying excited states with $E_\gamma < 0.5$ MeV → quantum tunneling properties) b) β-decay to excited states or b) nuclear reactions (high energy exciations)





- One photon de-excitation ↔ γ cascades
- 10 keV < $E_\gamma$ < 5 MeV and $10^{-16} < T_{1/2} < 10^{-8}$
- some exceptions: e.g. $^7$Li + p → $^8$Be + γ
  - $E_\gamma$ = 17 MeV (nucleon emission is forbidden by parity / angular momentum conservation)

# gamma decay

- Energy and momentum conservation: $\mathbf{p}_\gamma + \mathbf{p}_M = 0$ and $E_1 - E_2 = \Delta E = E_\gamma + E_M$
- Recoil energy of the nucleus:

(typically ~ 0.1 – 10 eV)

$$E_N = \frac{E_\gamma^2}{2 M_N c^2} \ll E_\gamma$$

Negligible. But not in Mössbauer spectroscopy!

- angular momentum conservation: selectrion rules

$$|I_1 - I_2| \equiv \Delta I \leqslant l \leqslant I_1 + I_2$$

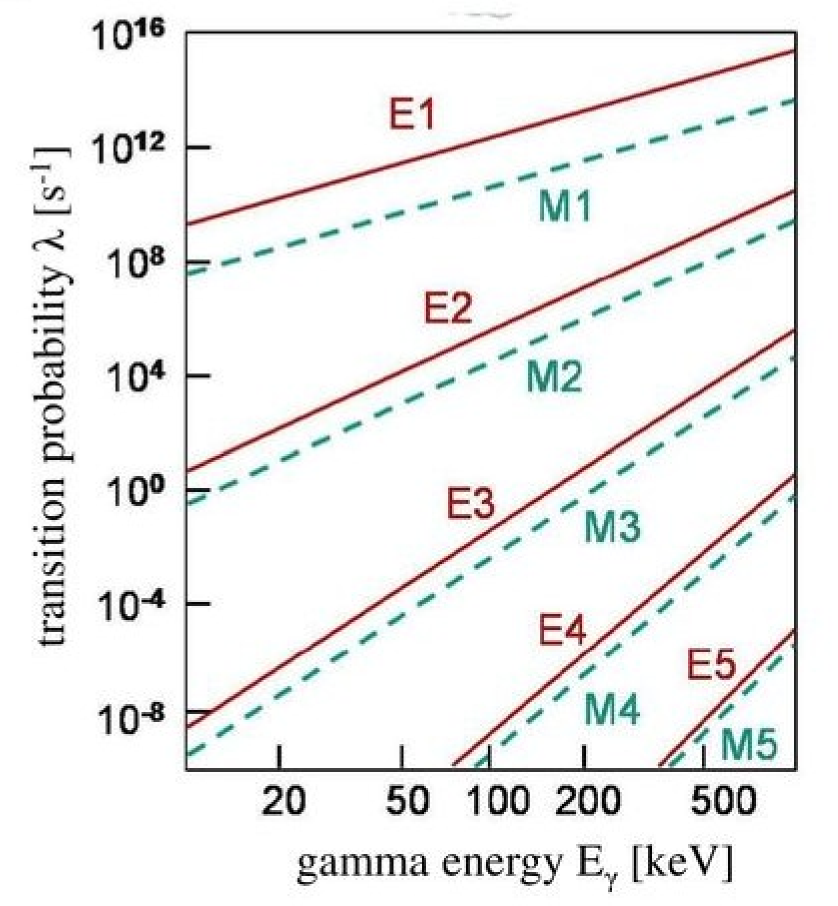
- l=1 dipole; l=2 quarupole; l=3 octupole radiation (no monopole radiation, photon spin is 1ħ) → multipolarity is from angular distribution
- polarisation: electric (E) and magnetic (M)
- parity conservation: $P_1/P_2=(-1)^l$ (electric transtion) and $P_1/P_2=(-1)^{l+1}$ (magnetic)

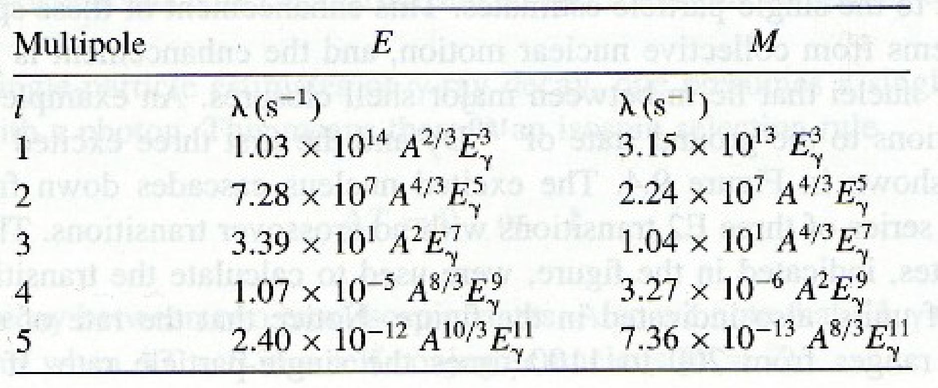
# gamma decay

- γ decay probability strongly depends on the *l* multipolarity and $E_\gamma$ transition energy:
  - increasing l → decreasing transition probability (by a factor of $10^{-4}$)
    - dipole and quadrupole radiations dominate
    - large spin-difference in nucleus → gamma decay probability is small (isomeric state)
    - transitions in a rotational band: cascades!
  - for given l magnetic transition probabyility is $10^2$-$10^3$ smaller than electric
- Weisskopf estimates: relation between $E_\gamma$ and $T_{1/2}$ for different multipolarities

$$|I_1 - I_2| \equiv \Delta I \leqslant l \leqslant I_1 + I_2$$

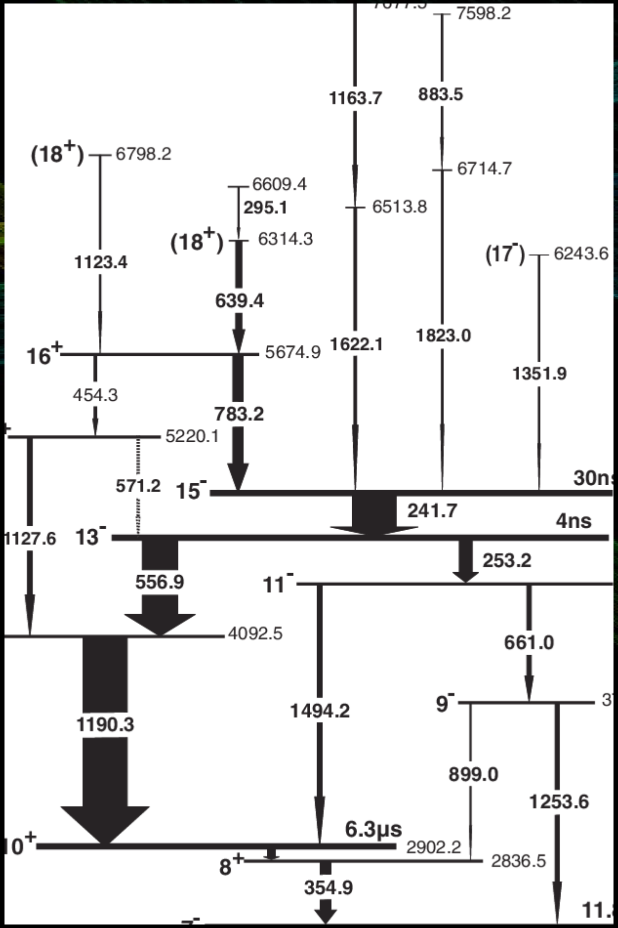
$$|I_1 - I_2| \equiv \Delta I \leqslant l \leqslant I_1 + I_2$$





| Multipole | $E$ | $M$ |
|---|---|---|
| $l$ | $\lambda\ (s^{-1})$ | $\lambda\ (s^{-1})$ |
| 1 | $1.03 \times 10^{14}\ A^{2/3} E_\gamma^3$ | $3.15 \times 10^{13}\ E_\gamma^3$ |
| 2 | $7.28 \times 10^{7}\ A^{4/3} E_\gamma^5$ | $2.24 \times 10^{7}\ A^{4/3} E_\gamma^5$ |
| 3 | $3.39 \times 10^{1}\ A^{2} E_\gamma^7$ | $1.04 \times 10^{1}\ A^{4/3} E_\gamma^7$ |
| 4 | $1.07 \times 10^{-5}\ A^{8/3} E_\gamma^9$ | $3.27 \times 10^{-6}\ A^{2} E_\gamma^9$ |
| 5 | $2.40 \times 10^{-12}\ A^{10/3} E_\gamma^{11}$ | $7.36 \times 10^{-13}\ A^{8/3} E_\gamma^{11}$ |

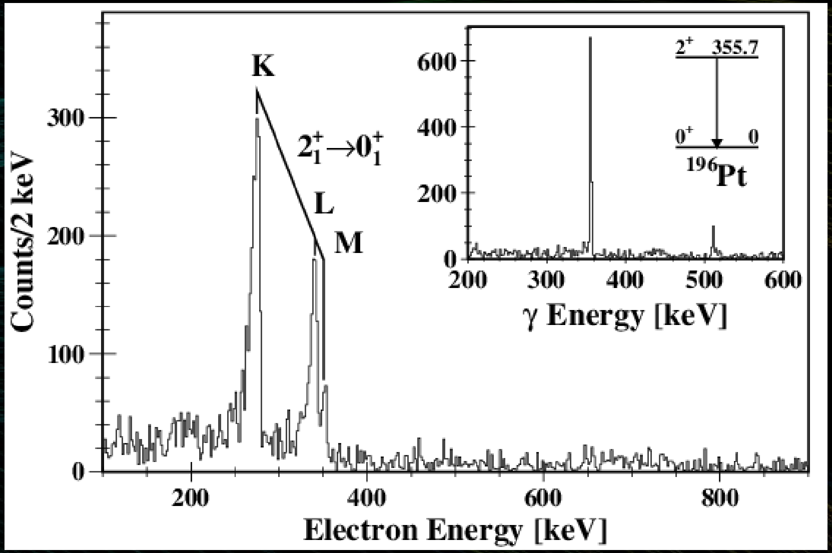
# gamma decay: level scheme



- arrows: thickness give the probability
- spin and parity, transition energies, level energy, branching ratios

# gamma decay

- Internal conversion: if gamma decay is forbidden (like isomeric states)
  - direct de-excitation by emitting a K electron
  - e.g $0^+ \rightarrow 0^+$ transition
  - internal conversion coefficients for K, L, M electrons: extracting multipolarity of the transitions!
  - $E_{e-} = E_\gamma - E_{K(LM)}$



- Internal pair creation:
  - electron-positron pairs are created
  - $E_{e-} + E_{e+} = E_\gamma - 1022$ keV

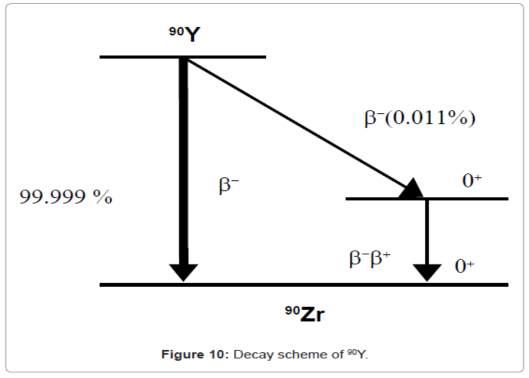
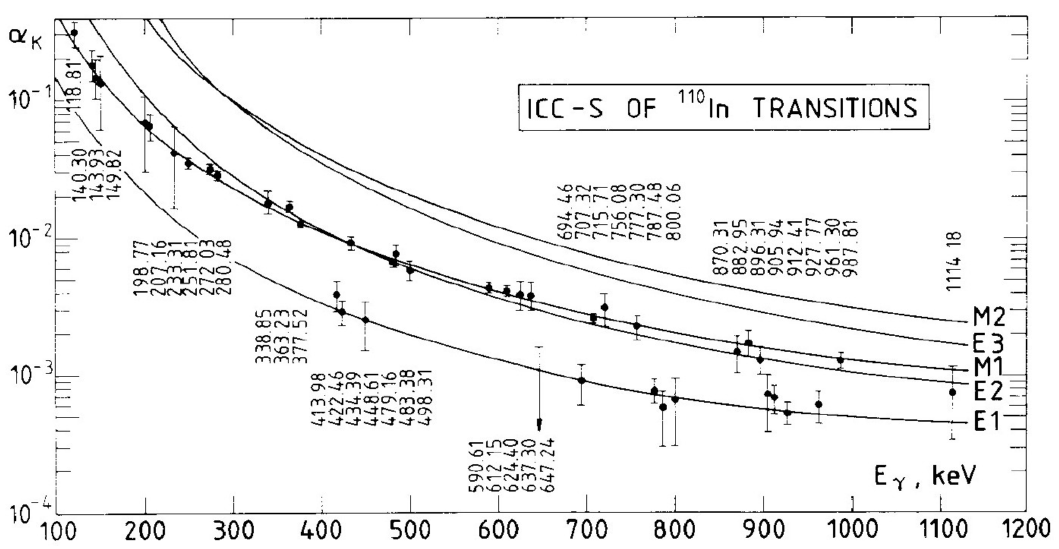
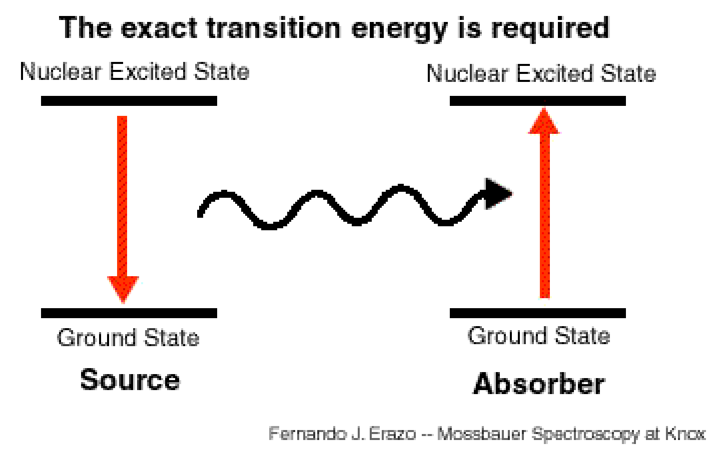


**Figure 10:** Decay scheme of $^{90}$Y.



# Mössbauer effect



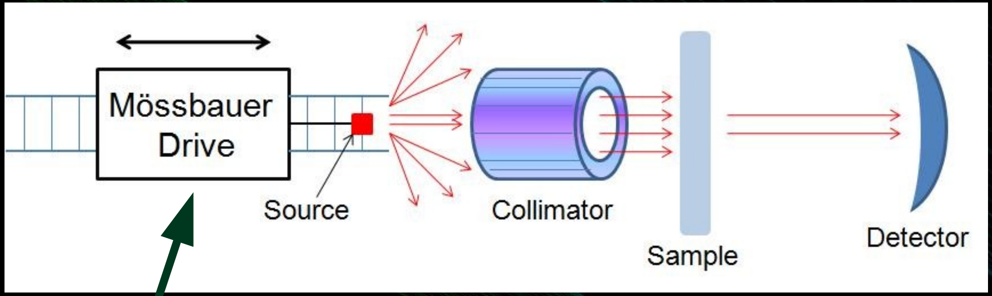
Fernando J. Erazo -- Mossbauer Spectroscopy at Knox

$$E_N = \frac{E_\gamma^2}{2 M_N c^2}$$

**due to momentum conservation**

- emitted $E_\gamma < E_{transition}$ while excitation needs a bit larger $E_\gamma$ → if identical nuclei are in rest and free, no absorbtion can be observed due to recoil

- If nuclei are in a solid crystal → phonons -vibrations of crystal lattice with discrete energy- can be emitted → if no phonon, then absorption and emission is recoilless due to large (M momentum conservation with a crystal as a whole)
- „recoilless nuclear resonance fluoresence”
- Mössbauer spectroscopy:
  - source and sample material is the same (as crystal)
  - source is moving with velocity *v* → Doppler shift



$$E_\gamma = E_{\gamma 0}\left(1 + \frac{v}{c}\right)$$

# Mössbauer spectroscopy

- A typical experiment: changing (scanning) the velocity of source with a linear motor → and see the resonance absorbtion lines → Intensity vs. velocity is measured (typically ±10-20 mm/s)
- Energy resolution is extremely good: up to $dE/E = 10^{-13}$ !!
- Isomer shift, quadrupole splitting and hyperfine splitting can be measured